# Research Methods for Economics and Related Studies

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**Abstract**

This workbook shows how to use economic theories, statistical and econometric methods for conducting research to find answers to puzzling issues in modern economies. How to test predications of models based on theoretical analysis from optimising models in micro or macro, finance or business related fields of economics using empirical evidence using basic econometric or statistical or applied general equilibrium or strategic analyses is discussed and illustrated. It is argued that a researcher need to be more open and comprehensive while thinking about alternative research techniques applicable to analysis of a particular issue under consideration. Aim of this workbook is to complement to the programme in order to achieve an excellence in research methods required for various fields including economics, finance, business, marketing or management in the academic environment of a Business School. Homeworks and assignments are integral part of this study programme.

*JEL Classification: C*

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# Contents

# Contents

# List of Tables

# 1 Introduction

Research is about finding new things and making original contribution to the literature. It can be theoretical or applied. Economic theory is process of thinking about the economy in terms cause-effects relationships among important variables. Theory provides analytical framework, such as demand and supply functions, investment and consumption or revenue and spending or export and import functions, abstracting away from the complexity of the real world.

Applied research aims to test certain hypothesis based on economic theories. Statistical and econometric methods are used for estimating parameters of a model and general equilibrium models to simulate the economy and to generate scenarios under available policy alternatives. Strategic interactions among economic agents are analysed applying game theoretical models. Public policies on taxes, spending, redistribution, trade, environment, labour and financial markets, research and development influence allocations of resources both by households and government. Economic research can show how much these economic agents are able to achieve their objectives given their constraints, what determines the most efficient allocation of resources in an economy and what is the best course of actions to maximise the social welfare.

### 1.0.1 Research Topics

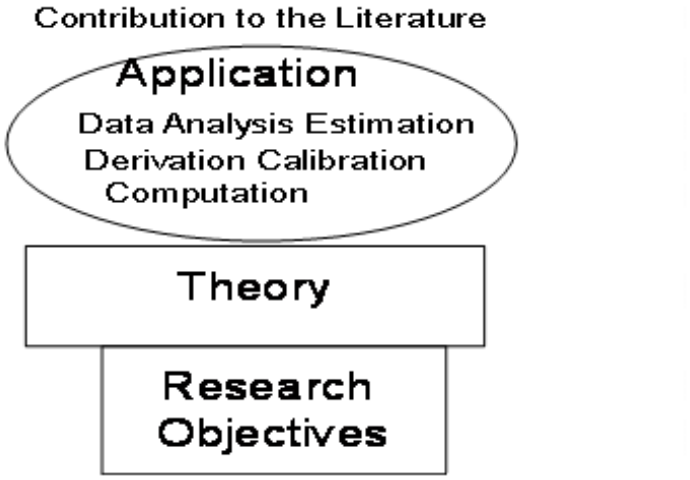
Starting point of any research is choice of a suitable topic.

1. Select a topic and justify why it is important to research on that topic.
2. Find out whether something has already been published in this topic or not. For such literature review see appropriate journals, books, reports, working papers or proceedings. Assess what sorts of research method were used by them.
3. Determine research method based on a thorough review of theoretical, analytical and empirical methods in the context of proposed topic.
4. Set up research questions clearly. Write a hypothesis concisely and test that empirically using available data.
5. Use primary or secondary data as appropriate. In secondary data look at cross sections surveys or time series or the panel of both.
6. State contribution to the literature made by the current research. Assess what has been achieved and what was anticipated.
7. Provide clear conclusions.

### 1.0.2 Research objectives

It is important to be clear about objectives of research. In microeconomics most common objective of a firm is to maximise profit, or revenue by producing commodities most efficiently. It is about increasing consumer and producer surplus and about surviving in the very competitive market. Objectives of households is to maximise their utility or welfare. A researcher might be interested to know these economic agents are doing (success or failure in business), in achieving their objectives based on cross section or time series evidences. Merger- acquisition, integration-disintegration, cartels, mark ups, average, fixed and marginal costs, elasticities of demand and supply with respect to

prices, income or tastes, economies of scale and scope, market concentration, distribution of income between capital and labour or among different categories of households are some examples. Statistical analysis is required to summarise data and get a sensible picture of the firms and households that may belong to various, industries, regions or situations. In macro economics major quests relate to long run economic growth, their determinants, technological progress and short run issues such as the fluctuations in output, employment, price level, interest rate, wage, exchange rate, money and credit, trade among various nations. How to raise the rate of economic growth, how to control inflation and reduce the unemployment rate? How to stabilise the aggregate demand and how to balance the budget, current and capital accounts.




Research objectives should be very specific. It should be enumerated if possible. Objectives should always be given priority in inductive and deductive reasoning, in formulating hypothesis and in conducting the literature review, choosing methods of analysis, or in collection of data, actual analysis, deriving the conclusion and recommendations. Objectives gives a focus to the research. It should justify why a research in a certain topic is urgent and important or what shorts of contribution can be made in the topic.

### 1.0.3 Literature review

It is a critical review on works done by others before. It is very difficult to find a topic where no one has done any work. However, it is possible that earlier author may have written in the different context, applied different methods or may have done only one part of it. Economic Journal is being published for more than 120 years; American Economic Review since 1911 and Econometrica since 1933. There are more than 775 journals listed in JSTOR database. Econlit is an electronic bibliography is a comprehensive index of journals, books, book reviews, journal, working papers and dissertations. Social Science Research Index (SSRN) lists working papers. What other people have done in a certain topic can be found searching the electronic databases. Thus the literature review provides a proper context for initiating the research. While write the literature review state how the earlier authors justified their research on that topic, what kind of theoretical and empirical methods did they apply and what sorts of conclusion did they arrive. Reflect whether those finding are still valid or not and in what way the proposed topics is going to add on to that knowledge.

### 1.0.4 Methodology for Research in Economics

Economists have developed many theories regarding how the various markets function or should function. How the various pieces of economic activities make the national or international economy. Economic research therefore is divided into two main groups 1) theoretical research 2) applied research. Theoretical research often involves derivation of demand supply equilibrium conditions using some sort of optimising process. Diagrams, equations or simply the logical statements are often used for theoretical deduction. Standard micro or macro economic models or extension of those in various fields like finance, engineering, environment, trade, public finance are applied to study optimisation by consumers and producers, determination of prices in a markets for goods and services or factors of production. The general equilibrium models quantify the entire economy. Intertemporal models show the process of accumulation, investment and growth. Statistically inferences based on marginal or cumulative distributions of populations, samples with law of large numbers are used to test claims of these theories. Abstract models require algebra, calculus, matrix, econometrics, real analysis or stochastic probability theory to represent and test these theoretical ideas.

Theories need to be applied in practice to make them useful for improvement in the welfare of human society. The application involves systematic collection of information on variables identified by the relevant theory. Empirical research tests the claims made by those theories stated in linear or non-linear functions using various estimation or computation techniques. As amount of information has grown so has the need to processing the information.

The applied research is basically about processing information consistently, coherently, systematically using inductive methods. Applied research can also vary according to the nature of method used in analysis. There are mainly four categories of applied research: 1) statistical and econometric analysis 2) calibration and computations of system of equations 3) strategic analysis 4) experimental analysis.

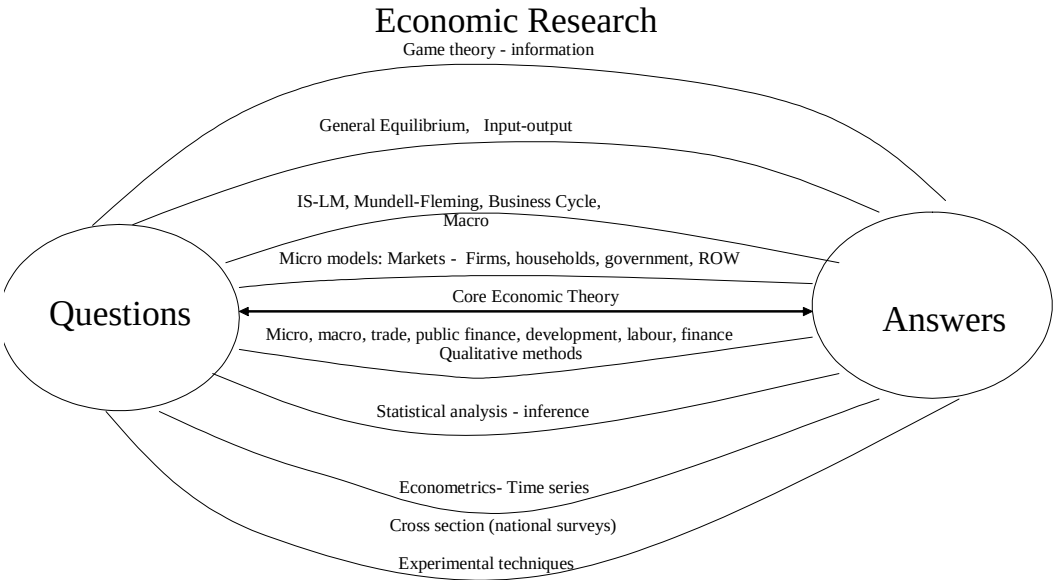
Statistical analysis involves designing, implementing and collecting data on economic variables scientifically in an unbiased manner. This also involves determining the properties of distribution of those variables, collecting information on central tendencies, finding correlations and the pattern of causality among variables. Econometric analysis involves techniques and applications to process data for testing various economic theories based on cross sections and time series data.

Calibration and computation of system of equations involves solving N number of equations on the basis of certain assumption about their behavior, such as market demand and supply functions, or input-output analysis or a general equilibrium system. Linear, non-linear or dynamic programming is often used to determine such a system.

Game theory is becoming increasingly popular tool to analyse inter-dependence among economic agents where the action to be taken by one is determined by the beliefs or perception of that individual about the action taken by other people in the economy. They are applied to analyse the process and outcome of bargaining, strategic contingency planning or just in describing the behaviors of economic agents. Experimental analysis has the concept of using control groups for testing economic theories, such as impacts of certain policy in economic stability, such as the adoption of euro, effect of certain drugs, or certain measures on productivity, health or educational attainment.

Aim of this research methods workbook is to introduce students to quantitative and analytical tools required to prepare their ability to write meaningful essays in other modules, develop a good research proposal for dissertations. It aims to provide basic skills required to execute research programs as a professional economist taking account of the most relevant economic theory and

available primary or secondary data set in economic issues.




Each of these techniques will be elaborated later on.

Most applied economics research require blending of a number of topics. Literature review is followed by theoretical analysis first and statistical, econometric or even computations exercises. Models should be as accurate and simple as possible.

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### 1.0.5 Guidelines for Empirical Research

1. Construction of hypotheses
   (a) Specify equations of a model based on economic theory.
      i. Macroeconomic model:
         A. determinants of growth
         B. total factor productivity
         C. growth, inequality and environment
         D. models of fluctuation or business cycle (interest rate rule)

E. determinants of consumption, saving, investment, exports, imports, output, employment
F. inflation and unemployment
G. interest rate, exchange rate, inflation, trade balance,
H. deposit expansion, credit

(b) Microeconomics model

i. Demand for a commodity (necessary, luxury or normal goods; durable or perishable items)
ii. Cost of production of certain commodity (car, plane, computer, TV, electronic goods, machines)
iii. Market price for a certain commodity (rice, wheat, maize, millet; meat and other livestock products)
iv. Profits, revenue of a certain company or industry (Barclays, British Airways, BT, Train/Bus, Low cost airlines)
v. Wage rates, rental rate of capital, salary of executives (job satisfaction and performance)
vi. structure of markets (competition, monopoly, oligopoly or monopolistic competition)
vii. foreign direct investment (joint ventures, franchising, licensing)
viii. merger and acquisition; economies of scale (concentration ratios)
ix. research and development; new management practices; business models
x. Efficiency and welfare

(c) Trade, prices and wages

(d) Public finance

i. Determinants of revenue and spending
ii. Budget deficit, public debt

(e) Environment

i. Determinants of pollution
ii. Global warming

(f) Employment and labour markets

i. Demand and supply of labour
ii. Employment/labour force
iii. Unemployment and inflation
iv. migration, labor force, population

(g) Finance

i. Optimal allocation, risk free return, return on financial assets
ii. Prices of stocks, bonds
iii. Foreign exchange market, commodity markets

(h) Economic development

i. Investment and growth
ii. Human capital and productivity
iii. Structural transformation
iv. Estimation of surplus labour
v. Gini coefficient

Theoretical derivation requires using first and second order conditions and solving a system of equations of a model; survey experiments.

2. Preparation of data

Once clear about the hypothesis, required data can be obtained by primary survey or downloaded from standard secondary sources like

1. (a) Economic and social data (ESDS) www.esds.ac.uk/international
   (b) http://libguides.hull.ac.uk/skillsguides
   (c) datastream: http://banker.thomsonib.com/ta/?ExpressCode=Hull
   (d) Eurostat: http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/themes
   (e) BHPS: http://www.esds.ac.uk/government/; http://www.esds.ac.uk/government/surveys/
   (f) http://www.data-archive.ac.uk/findingData/bhpsTitles.asp
   (g) http://www.statistics.gov.uk
   (h) http://www.economicsnetwork.ac.uk/links/data_free#uk
   (i) http://www2.hull.ac.uk/acs/ict/software/software_sales.aspx
   (j) http://www2.hull.ac.uk/student/studyadvice.aspx
   (k) http://www2.hull.ac.uk/student/studyadvice/mathematicsresources.aspx

3. Data files

   (a) Excel or CSV format for PcGive; Eviews.
   (b) Large scale data are available on SPSS or
   (c) STATA format (*.dat).

You can get lost at this stage if you are not precise on the hypothesis. Be focused and get only data you require. Ask for help if confused.

4. Estimations and interpretation of results

   (a) Specify equations according to the hypothesis set in stage 1
   (b) Derive equations for the OLS estimators
   (c) Examine the properties of those estimators - do they have expected signs, are they significant? what do they mean?
   (d) Expected values and variance of those estimators; reliability and confidence interval for estimated parameters.

(e) Estimate parameters using the data collected above.

5. Analyse variance by $R^2$, $t$, $F$ and $\chi^2$ tests as appropriate

   (a) Write analytical forms for $t$, $F$ and $\chi^2$ tests
   (b) Determine the degrees of freedom and critical values from the theoretical tables.
   (c) Compare empirical results with those theoretical values and determine the significance.
   (d) Take a decision regarding significance of the model or coefficient based on these tests
   (e) Think of improving the model based on tests.

6. Tests of multicollinearity

   (a) Write estimators at least with two explanatory variables
   (b) Show how the estimator breaks down with perfect multicollinearity
   (c) Determine variables that are correlated to each other based on analysis of correlation among explanatory variables
   (d) Determine the variance inflation factor
   (e) Drops correlated variables and re-estimate the model until getting the sensible results.

7. Restrictions and dummy variables

   (a) Consider theoretically appropriate restrictions in the model.
   (b) Write analytical forms of F-test that can be used to test restrictions.
   (c) Determine the validity of restrictions.
   (d) Introduce dummy variables to capture structural changes in time series or individual effects in the cross section Analysis.

8. Heteroskedasticity

   (a) Write the analytical form of the heteroskedasticity problem.
   (b) Show how the properties of the OLS estimators are affected by presence of heteroskedasticity
   (c) Write test statistics to detect heteroskedasticity
   (d) Transform the model to remove heteroskedasticity
   (e) Construct a cross section data appropriate for heteroskedasticity analysis
   (f) Interpreted meaning of the heteroskedasticity consistent standard errors

9. Autocorrelation

   (a) Find causes why there is autocorrelation and consequences of it
   (b) Write analytical form of the Durbin-Watson Statistics
   (c) Show how the properties of the OLS estimators are affected by presence of autocorrelation

(d) Estimate the model with AR(1) autocorrelation

(e) Transform the model to remove autocorrelation using iterative procedure

10. Stationarity

(a) Explain when a variable is stationary and when non-stationary.

(b) Show the impact of non-Stationarity in the variance of the variable in an AR(1) model

(c) What is Dickey-Fuller test and Augmented Dickey-Fuller test? Phillip-Perron tests.

(d) Determine whether your series are stationary based on DF and ADF statistics.

11. Causality and cointegration

(a) Show the procedure for Granger causality test.

(b) What is order of integration and what is cointegration?

(c) Show analytical forms to test cointegration

(d) Determine cointegration in a single equation model.

12. Write an essay or article based on research experience gained in following steps 1 to 11.

### 1.0.6 Deadlines

Most often research need to be conducted within certain deadline. Gantt chart shows allocation of time to research activities.

| Tasks | June | July | August | September |
|---|---|---|---|---|
| | | | | |
| Proposal | | | | |
| | | | | |
| Literature review | | | | |
| Model and Analytical Structure | | | | |
| Design of survey | | | | |
| Survey/Data collection | | | | |
| | | | | |
| Analysis | | | | |
| | | | | |
| Estimation and Tabulation | | | | |
| | | | | |
| Writing the first complete draft | | | | |
| | | | | |
| Presentation and revision | | | | |
| | | | | |
| Final submission | | | | |

### 1.1 Homework 1 (due next week)

1. What is your research objective and topic?
2. List and review any five academic articles published in this topic.
3. What methods have been used in the literature? How will you conduct the research?
4. Estimate tentative cost of research breaking down its components man-hours, materials and other items.
5. What is your expected contribution?

## 2 Microeconomic Modelling and Programming

Think about a sample of microeconomic topics such as:

1. Demand for commodities: food, housing, and clothing items; domestic kitchen appliances; durable goods cars; utilities, services telephone, banking, insurance
2. Income and substitution effects of price changes
3. Consumption patterns by level of income
4. Equivalent and compensating variances of price changes
5. Determinants of consumption and saving
6. Analysis of short and long run cost of a certain firm or industry
7. Consequences of factor and product taxes in a competitive market
8. Impacts of new technology in costs of production and supply
9. Competition and efficiency
10. Market imperfections, inefficiency and regulation
11. Factor prices: wage, interest rates, profits
12. Stock prices
13. Welfare consequences of duopoly or oligopoly in markets: energy, telecom, airlines
14. Impacts of regulation and privatisation on efficiency of production
15. Affluence and poverty: analysis of expenditure pattern of households
16. Elasticities of supply and demand and deadweight loss consumers or producers
17. Examination of benefits and costs of privatisation

18. Uncertainty, externalities

19. FDI, national and multinational corporations

Microeconomic theories that explain behavior of households and firms are appropriate for any topics like above. These theories could be presented in a set of relevant diagrams and equations. Basic analytical models should be simple. As research progresses it requires use of statistical or econometric estimations and simulations which are easily done using standard software.

Commodity Markets:

1. agricultural goods: sugar, potato, cotton, rubber, green vegetables, tomato

2. fruit: apples, banana, pears, grapes, oranges, mango, jackfruit , coconut, nuts

3. grains: rice, corn, millet, wheat, maize, palm oil, peanuts

4. meat market: fish, beef, pork, lamb

5. drinks: wine, beer, whiskey, martini

6. metals and minerals: gold, silver, aluminium, steel, iron, copper, tin, zinc, oil

Research objective: Major aim is to study consumer behavior, e.g. what determines spending on daily necessities ($x_1$) and other items ($x_2$).

When preferences are Cobb-Douglas, consumers' problem with $\alpha + \beta = 1$is

$$max \quad u(x_1, x_2) = x_1^{\alpha} x_2^{\beta} \tag{1}$$

subject to:

$$p_1 x_1 + p_2 x_2 = m \tag{2}$$

$$L = x_1^{\alpha} x_2^{\beta} + \lambda \left[ m - p_1 x_1 - p_2 x_2 \right] \tag{3}$$

1.

$$\frac{\partial L}{\partial x_1} = \alpha x_1^{\alpha - 1} x_2^{\beta} - \lambda p_1 = 0 \tag{4}$$

$$\frac{\partial L}{\partial x_2} = \beta x_1^{\alpha} x_2^{\beta - 1} - \lambda p_2 = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = m - p_1 x_1 - p_2 x_2 = 0 \tag{6}$$

Solving this one get demand for goods $x_1$ and $x_2$

$$x_1 = \frac{\alpha m}{p_1}; \qquad x_2 = \frac{\alpha m}{p_2} \tag{7}$$

What does this demand function imply:

2. demand increases when income rise and falls when price rise

Take logs of the demand function

$$\ln(x_1) = \ln\alpha + \beta_1 \ln m - \beta_2 \ln p_1 \tag{8}$$

$$\ln(x_2) = \ln(1-\alpha) + \gamma_1 \ln m - \gamma_2 \ln p_2 \tag{9}$$

This is a demand function that solves the consumers' optimisation problem. Now the empirical issue of research is to estimate $\alpha, \beta_1$ and $\beta_2$ parameters and the elasticity of demand.

Utility of household

$$V(x_1, x_2) = \left(\frac{\alpha m}{p_1}\right)^{\alpha} \left(\frac{\beta m}{p_2}\right)^{\beta} = m\alpha^{\alpha}\beta^{\beta}p_1^{-\alpha}p_2^{-\beta} \tag{10}$$

Research question: what is the impact on utility of changes in prices due to changes in taxes, or tastes or some exogenous income. These can be done finding equivalent and compensating variances of price changes.

### 2.0.1 Consumer and producer surplus

$$P^d = 25 - Q^2 \tag{11}$$

$$P^s = 1 + 2Q \tag{12}$$

Demand supply balance

$$25 - Q^2 = 1 + 2Q \tag{13}$$

$$Q^2 + 2Q - 24 = 0 \tag{14}$$

$$Q^2 + 6Q - 4Q - 24 = Q(Q+6) - 4(Q+6) = 0 \tag{15}$$

Either $Q = -6$ *or* $Q = 4$
$P = 25 - Q^2 = 25 - 4^2 = 9$
Consumer surplus

$$\begin{aligned} CS &= \int_0^4 \left(25 - Q^2\right) dQ - PQ = \left(25Q - \frac{Q^3}{3}\right|_0^4 - PQ \\ &= 100 - \frac{64}{3} - 36 = \frac{128}{3} = 42.7 \end{aligned} \tag{16}$$

Producer surplus

$$PS = PQ - \int_0^4 (1+2Q)\,dQ = PQ - ((1+2Q)\,dQ|_0^4$$
$$= PQ - \left(\frac{2Q^2}{2} - Q\right|_0^4 = 36 - \frac{32}{2} - 4 = 36 - 20 = 16 \quad (17)$$

### 2.0.2 Profit

Estimating a revenue, cost and profit functions of a certain corporation:

Revenue:

$$R = aQ - bQ^2 \quad (18)$$

Cost:

$$C = dQ^3 - eQ^2 + fQ + 845 \quad (19)$$

Profit:

$$\Pi = R - C = (a-f)\,Q - (b+e)\,Q^2 - dQ^3 - 845 \quad (20)$$

Now get the data on Q, R, C then parameters $a, b, c, d, e, f$ can be estimated using econometric techniques. Calculate them and put them in a diagram. This model can be applied then to estimate or predict profits. If parameters are a = 5900; b = 10; d = 2; e = 4; f =140; n =845; then show that Q =30; R=168000 and C =55445.

[hint use MR =MC or $\frac{\partial R}{\partial Q} = \frac{\partial C}{\partial Q}$ or $\frac{\partial}{\partial Q} = 0$.

**GAMS Programme Code: Example** Parameters a, b, d, e, f, n;

```
Variable R, Q, C, P;
a = 5900; b = 10; d = 2; e = 4;
f =140; n =845;
Equations ER, EC, EP;
ER..
R =e= a*Q-b*Q**2;
EC..
C =e= d*Q**3-e*Q**2+f*Q+n;
EP..
P =e= R-C;
model cost /all/;
R.lo=1;
Q.lo =1;
C.lo=0.01;
Solve cost maximising P using nlp;
Display R.l, Q.L, C.l;
```

## 2.1 A general equilibrium model

What is general equilibrium

- Households and firm optimise subject to their constraints
  - Utility maximisation by households and profit maximisation by firms
- System of prices when all markets clear simultaneously (all goods and factor markets)

$$D\left(p_1\ p_2\ \ p_3, ....p_n\right) = S\left(p_1\ p_2\ \ p_3, ....p_n\right) \tag{21}$$

  Excess demand is zero in equilibrium.

- Income of agents equals their expenditure
- Imports equals exports in an open economy model
- Saving equals investment in a dynamic economy model
- Public spending accounts are balanced in model with public sector
- General equilibrium is obtained by the price system when economy is in perfect harmony.
- Consider one of the easiest possible example of a general equilibrium model with production
- Household gets utility from consuming goods and leisure

$$\underset{c,l}{Max}\ U = C^{\alpha} L^{(1-\alpha)} \tag{22}$$

  Subject to

$$p.C + w.L_h = w\overline{L} \tag{23}$$

  Lagrangian optimisation:

$$L\left(C, L_h, \lambda\right) = C^{\alpha} L^{(1-\alpha)} + \lambda\left[w\overline{L} - p.C - w.L_h\right] \tag{24}$$

- Optimal demand for goods $C$.solving the first order conditions

$$C = \frac{\alpha w\overline{L}}{p} = \frac{\alpha\overline{L}}{\frac{p}{w}} \tag{25}$$

  Households buy more when goods are cheaper and when they have more income

- Optimal demand for leisure $L_h$

$$.L_h = \frac{\left(1-\alpha\right) w\overline{L}}{w} = \left(1-\alpha\right)\overline{L} \tag{26}$$

- if $\overline{L} = 1600$ and $\alpha = 0.4$ then $.L_h = 0.6 \times 1600 = 960$.

Firms' Problem: maximise profit

$$\Pi = PY - w \cdot L_f \tag{27}$$

$$Y = L_f^{\beta} \tag{28}$$

$$L_f = \left(\frac{\beta P}{w}\right)^{\frac{1}{1-\beta}} \tag{29}$$

$$Y = \left(\frac{\beta P}{w}\right)^{\frac{\beta}{1-\beta}} \tag{30}$$

Let $\beta = 0.5$
Clearing Goods and Labour Markets: Real Wage Rate

$$Y = C \tag{31}$$

$$L_f + L_h = \overline{L}; \quad L_f = 1600 - 960 = 640 \tag{32}$$

$$Y = \left(\frac{\beta P}{w}\right)^{\frac{\beta}{1-\beta}} = 640^{0.5} = 25.29 \tag{33}$$

$$C = \frac{\alpha \overline{L}}{\frac{p}{w}} = \frac{0.4 \times 1600}{\frac{p}{w}} = y = 25.29 \tag{34}$$

$$\frac{p}{w} = \frac{0.4 \times 1600}{25.29} = 25.29 \tag{35}$$

if $w = 1$ set as numeraire labour market clears as

$$L_f + L_h = 640 + 960 = 1600 = \overline{L} \tag{36}$$

Parameters and shadow prices

Table 1: Parameters of the General Equilibrium Model

| Parameters | Value |
|---|---|
| $\alpha$ | 0.4 |
| $\beta$ | 0.5 |
| $\overline{L}$ | 1600 |
| $w\,(normalised)$ | 1 |

$$\lambda = \frac{\alpha \left(\frac{L_h}{C}\right)^{0.6}}{p} = \frac{0.4 \left(\frac{640}{25.29}\right)^{0.6}}{25.29} = 0.12 \tag{37}$$

Shadow price in tax scenario

$$\lambda_T = \frac{\alpha\left(\frac{L_h}{C}\right)^{0.6}}{p} = \frac{0.4\left(\frac{480}{21.90}\right)^{0.6}}{21.90} = 0.116 \tag{38}$$

This is the change in utility associated to unit change in income.

Allocations and Prices in Equilibrium

Table 2: General Equilibrium Solutions

| Variable | Base No Tax Solution | Tax Solution |
|---|---|---|
| output $(Y)$ | 25.29 | 21.90 |
| Consumption$(C)$ | 25.29 | 21.90 |
| Leisure$(L_h)$ | 960 | 720 |
| Labour demand$(L_f)$ | 640 | 480 |
| Utility$(U)$ | 224.19 | 178.09 |
| Relative price$\left(\frac{p}{w}\right)$ | 25.29 | 21.90 |
| Shadow Price | 0.12 | 0.116 |

Welfare loss to households from the government $= (224.19 - 178.09)/224.19 = 0.2056 = 20.56\%$.

Effective labour tax = 400/1600=0.25= 25%. True if households do not get utility of from public spending. How far this is true depends on the efficiency of the public sector.

**GAMS Code** $Title tax in leisure consumption model

```
parameters a, b, lbar, t;
a =0.5; b=0.6; lbar =1600; t = 0.15;
variables u, c, l, ls, p, w, y, G, R;
equations utils, EC, EL, ELS, EY, MKG,ER,EG;
utils..
U =e= C*L;
EC..
C =e= (1/2)*(w*lbar/(p*(1+t)));
EL..
L =e= (p*(1+t)/w)*C;
ELS..
LS =e= lbar - L;
EY..
Y =e= LS**(1/2);
MKG..
Y =e= C;
ER ..
R =e= t*P*C;
EG..
G=e= R;
u.lo=0.10;
c.lo=0.10;
l.lo=0.10;
ls.lo=0.10;
```

p.lo=0.10;
w.lo=0.10;
y.lo=0.10;
G.lo=0.10;
R.lo=0.10;
model TCL /all/;
solve TCL maximising U using nlp;
Decomposition of income and substitution effects of tax changes

$$Max \;\; U = X_1^{0.4} X_2^{0.6} \tag{39}$$

- Subject to

$$p_1.X_1 + p_2.X_2 = 150 \tag{40}$$

Lagrangian optimisation:

$$L\left(X_1, X_2, \lambda\right) = X_1^{0.4} X^{0.6} + \lambda\left[150 - p_1.X_1 - p_2.X_2\right] \tag{41}$$

- For base equilibrium assume that $p_1 = 3$ and $p_2 = 2$.
- Optimal demand for goods $X_1$

$$X_1 = \frac{0.4\left(150\right)}{p_1} = \frac{60}{3} = 20; \quad X_2 = \frac{0.6\left(150\right)}{p_2} = \frac{90}{2} = 45 \tag{42}$$

$$U_0 = X_1^{0.4} X_2^{0.6} = \left(20\right)^{0.4}\left(45\right)^{0.6} = 32.53 \tag{43}$$

Now assume that there is a subsidy in $X_1$ of £1 and price reduces from 3 to 2; $p_1 = 2$.

Equivalent Variation

What is the Hicksian Equivalent and compensating variations of price change? What are the income and substitution effects of this price change?

First find out how much money is required at new prices to guarantee the original utility by solving

$$U_0 = \left(\frac{0.4\left(m'\right)}{2}\right)^{0.4}\left(\frac{0.6\left(m'\right)}{2}\right)^{0.6} = 32.53 \tag{44}$$

$$U_0 = \left(\frac{0.4\left(m'\right)}{2}\right)^{0.4}\left(\frac{0.6\left(m'\right)}{2}\right)^{0.6}; \;\; m' = \frac{2\left(32.53\right)}{0.4^{0.4} \times 0.6^{0.6}} = 127.49 \tag{45}$$

- Equivalent variation (money to be taken away when prices fall)

$$EV = 150 - 127 - 49 = 22.51 \tag{46}$$

Compensating Variation

- For compensating variation first compute the demand in new prices and utility

$$X_1 = \frac{0.4\,(150)}{p_1} = \frac{60}{2} = 30; \quad X_2 = \frac{0.6\,(150)}{p_2} = \frac{90}{2} = 45 \tag{47}$$

$$U_1 = X_1^{0.4} X_2^{0.6} = (30)^{0.4}\,(45)^{0.6} = 38.26 \tag{48}$$

$$U_1 = \left(\frac{0.4\left(m^{''}\right)}{3}\right)^{0.4} \left(\frac{0.6\,(m'')}{2}\right)^{0.6} = 38.26 \tag{49}$$

$$m^{'\prime} = \frac{2\,(38.26) \times 3^{0.4} \times 2^{0.6}}{0.4^{0.4} \times 0.6^{0.6}} = 176.39 \tag{50}$$

$$CV = 150 - 176.39 = -26.39 \tag{51}$$

Summarising the Money Metric Utility Changes Due to Taxes

Table 3: Summary of Equivalent and Compensating Variation

| | Fall in Price | Rise in Price | Fall in Price | Basis of evaluation |
|---|---|---|---|---|
| EV | + | - | 22.51 | New Price-Old Utility |
| CV | - | + | -26.39 | OLD Price- New Utility |

**Substitution Effect : 2.5; Income effect:7.6 and total effect: 10.**

This is partial equilibrium result - general equilibrium impacts must take interaction with all other markets. Ultimate impact can be much higher or much lower than this. It need to bring production, income distribution sides into account.

## 2.2 Burden of taxes in partial equilibrium models

Burden of Taxes in Partial Equilibrium Analysis (it depends on elasticities)

**Consider linear demand and supply model**

$$D = 150 - 3P \tag{52}$$

$$S = 30 + 2P \tag{53}$$

**Equilibrium D =S implies P=24 and Q = 78.**

Now there is tax in commodity so that consumers pay more and suppliers get less.

$$P^D = P^S + t \tag{54}$$

where $t$ is tax imposed per unit. Let $t = 2$.

$$D = 150 - 3P^D = 150 - 3\left(P^S + 2\right) \tag{55}$$

Burden of Taxes in Partial Equilibrium Analysis (it depends on elasticities)

$$D = 150 - 3P^D = 150 - 3\left(P^S + 2\right) \tag{56}$$

$$S = 30 + 2P^S \tag{57}$$

$P^D = 24 \quad P^S = 22.8$ Q= 75.6

Deadweight loss of taxes =loss of consumer surplus+loss of producer surplus=

$= 0.5(0.8\times2.4) + 0.5\,(1.2 \times 2.4) = 0.96 + 1.44 = 2.4$

Elasticity of demand = -3$\times\frac{24}{78} = 0.92$ Elasticity of supply = 2$\times\frac{24}{78} = 0.61$ . Thus more burden is taken by producers.

**General equilibrium impacts are much higher than the partial equilibrium impacts. You can compute a general equilibrium tax model using GAMS for more realistic economy using input-output tables presented last week.**

Demo version can solve only small models but not the large ones.

### 2.2.1 Interdependency among markets

After estimating slopes and intercepts of demand and supply functions the equilibrium prices and quantities could be found by solving the simultaneous equation system as:

Market 1:

$$X_1^d = 10 - 2p_1 + p_2 \tag{58}$$

$$X_1^S = -2 + 3p_1 \tag{59}$$

Market 2:

$$X_2^d = 15 + p_1 - p_2 \tag{60}$$

$$X_2^S = -1 + 2p_2 \tag{61}$$

Equilibrium in both markets implies:

$X_1^d = X_1^S$ implies $10 - 2p_1 + p_2 = -2 + 3p_1$
$X_1^d = X_1^S$ implies $15 + p_1 - p_2 = -1 + 2p_2$
This in matrix notation:

$$\begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 16 \end{bmatrix} \tag{62}$$

Application of Matrix in Solving Equations

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ -1 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 16 \end{bmatrix} \tag{63}$$

Determinant

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix} = (5 \times 3 - (-1)(-1)) = 15 - 1 = 14;$$

Cofactor transpose:

$$C' = \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}' = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$$

Solution by matrix inversion:

$$\begin{aligned} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} &= \frac{1}{14} \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 12 \\ 16 \end{bmatrix} \\ &= \frac{1}{14} \begin{pmatrix} (3 \times 12) + (1 \times 16) \\ (1 \times 12) + (5 \times 16) \end{pmatrix} = \begin{pmatrix} \frac{52}{14} \\ \frac{92}{14} \end{pmatrix} = \begin{pmatrix} \frac{26}{7} \\ \frac{46}{7} \end{pmatrix} \end{aligned} \tag{64}$$

**Cramer's Rule is easier**

$$p1 = \frac{\begin{vmatrix} 12 & -1 \\ 16 & 3 \end{vmatrix}}{\begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{36 + 16}{15 - 1} = \frac{26}{7}; \quad p2 = \frac{\begin{vmatrix} 5 & 12 \\ -1 & 16 \end{vmatrix}}{\begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{80 + 12}{15 - 1} = \frac{46}{7} \tag{65}$$

Market 1:

$$LHS = 10 - 2p_1 + p_2 = 10 - 2\left(\frac{26}{7}\right) + \left(\frac{46}{7}\right) = \frac{64}{7} = -2 + 3p_1 = \frac{64}{7} = RHS \tag{66}$$

Market 2:

$$LHS = 15 + p_1 - p_2 = 15 + \frac{26}{7} - \frac{46}{7} = \frac{85}{7} = -1 + 2p_2 = \frac{85}{7} = RHS \tag{67}$$

QED.

Extension to N-markets is obvious; a confidence for solving large models.

## 2.3 Input-output Model

Structure of an input-output table (snap-shot of the economy for a given time)

$$\begin{bmatrix} IO & F \\ VA & Trasfers \end{bmatrix} \tag{68}$$

Leontief coefficients

Input-Output Model: Structural Equations

$$X_1 = X_{11} + X_{12} + F_1 \tag{69}$$

$$X_2 = X_{21} + X_{22} + F_2 \tag{70}$$

$$a_{11} = \frac{X_{11}}{X_1}; \; a_{12} = \frac{X_{12}}{X_2}; a_{21} = \frac{X_{21}}{X_1}; \; a_{22} = \frac{X_{22}}{X_2}; \tag{71}$$

Table 4: Leontief Coefficients

| | Intermediate demand | | Final Demand | Total |
|---|---|---|---|---|
| | $X_1$ | $X_2$ | $F$ | $Y$ |
| $X_1$ | 10 | 20 | 70 | 100 |
| $X_2$ | 30 | 20 | 150 | 200 |
| Labour input | 40 | 50 | | 90 |
| Capital input | 20 | 110 | | 130 |
| Total | 100 | 200 | 220 | |

Table 5: Leontief Technology and Primary Input Coefficients

| | Intermediate demand | |
|---|---|---|
| | $X_1$ | $X_2$ |
| $X_1$ | 0.1 | 0.1 |
| $X_2$ | 0.3 | 0.1 |
| Labour input | 0.4 | 0.25 |
| Capital input | 0.2 | 0.55 |
| Total | 1.0 | 1.0 |

$$X_1 = a_{11}X_1 + a_{12}X_2 + F_1 \tag{72}$$

$$X_2 = a_{21}X_1 + a_{22}X_2 + F_2 \tag{73}$$

Input-Output Model

$$X_1 - a_{11}X_1 - a_{12}X_2 = F_1 \tag{74}$$

$$-a_{21}X_1 + X_2 - a_{22}X_2 = F_2 \tag{75}$$

$$\begin{bmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{bmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{76}$$

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{bmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{bmatrix}^{-1} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} \tag{77}$$

$$X = (I - A)^{-1} F \tag{78}$$

Solution of the input - output model by Cramer's Rule

$$|A| = \begin{vmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{vmatrix} = \left(1 - a_{1,1}\right) \times \left(1 - a_{2,2}\right) - a_{21}a_{12} \tag{79}$$

$$X_1 = \frac{\begin{vmatrix} F_1 & -a_{12} \\ F_2 & (1-a_{22}) \end{vmatrix}}{\begin{vmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{vmatrix}} = \frac{F_1(1-a_{22}) + a_{12}F_2}{\left(1-a_{_{1,1}}\right) \times \left(1-a_{_{2,2}}\right) - a_{21}a_{12}} \tag{80}$$

$$X_2 = \frac{\begin{vmatrix} (1-a_{11}) & F_1 \\ -a_{21} & F_2 \end{vmatrix}}{\begin{vmatrix} (1-a_{11}) & -a_{12} \\ -a_{21} & (1-a_{22}) \end{vmatrix}} = \frac{F_2(1-a_{11}) + a_{21}F_1}{\left(1-a_{_{1,1}}\right) \times \left(1-a_{_{2,2}}\right) - a_{21}a_{12}} \tag{81}$$

Numerical Example of Input Output Model

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{bmatrix} (1-0.1) & -0.1 \\ -0.3 & (1-0.1) \end{bmatrix}^{-1} \begin{pmatrix} 70 \\ 150 \end{pmatrix} \tag{82}$$

$$X_1 = \frac{\begin{vmatrix} 70 & -0.1 \\ 150 & 0.9 \end{vmatrix}}{\begin{vmatrix} 0.9 & -0.1 \\ -0.3 & 0.9 \end{vmatrix}} = \frac{63+15}{0.81-0.03} = \frac{78}{0.78} = 100 \tag{83}$$

Numerical Example of Input Output Model

$$X_2 = \frac{\begin{vmatrix} 0.9 & 70 \\ -0.3 & 150 \end{vmatrix}}{\begin{vmatrix} 0.9 & -0.1 \\ -0.3 & 0.9 \end{vmatrix}} = \frac{135+21}{0.81-0.03} = \frac{156}{0.78} = 200 \tag{84}$$

Solutions reproduce the benchmark data. Model is calibrated.

## 2.4 Review on Matrix

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$; $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$; $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$;

**Addition:**

$$A + B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix} \tag{85}$$

**Subtraction:**

$$A - B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}-b_{11} & a_{12}-b_{12} \\ a_{21}-b_{21} & a_{22}-b_{22} \end{bmatrix} \tag{86}$$

**Multiplication:**

$$AB = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{bmatrix} \tag{87}$$

Algebra

### 2.4.1 Determinant and Transpose of a Matrix

Determinant of $A$ (difference of cross products)

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (a_{11}a_{22} - a_{21}a_{12}); \tag{88}$$

Determinant of $B$ $|B| = \begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = (b_{11}b_{22} - b_{21}b_{12})$

Determinant of $C$ $|C| = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} = (c_{11}c_{22} - c_{21}c_{12})$

Transposes of A, B and C (interchange of rows to columns and columns to rows)

$$A' = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}; \; B' = \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix}; \; C' = \begin{bmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{bmatrix} \tag{89}$$

Singular matrix $|D| = 0$. non-singular matrix $|D| \neq 0$.

### 2.4.2 Inverse of A

$$A^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{|A|} adj\,(A) \tag{90}$$

$$adj\,(A) = C' \tag{91}$$

For $C$ cofactor matrix. For this cross the row and column corresponding to an element and multiply by $(-1)^{i+j}$

$$C = \begin{bmatrix} |a_{22}| & -|a_{21}| \\ -|a_{12}| & |a_{11}| \end{bmatrix} = \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix} \tag{92}$$

$$C' = \begin{bmatrix} a_{22} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}' = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \tag{93}$$

Inverse of A

$$\begin{aligned} A^{-1} &= \frac{1}{(a_{11}a_{22} - a_{21}a_{12})} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} \\ &= \begin{bmatrix} \frac{a_{22}}{(a_{11}a_{22} - a_{21}a_{12})} & -\frac{a_{12}}{(a_{11}a_{22} - a_{21}a_{12})} \\ -\frac{a_{21}}{(a_{11}a_{22} - a_{21}a_{12})} & \frac{a_{11}}{(a_{11}a_{22} - a_{21}a_{12})} \end{bmatrix} \end{aligned} \tag{94}$$

Exercise on matrix manipulations

1) Find $B^{-1}$.

## 2.5 Linear Programming

Linear Programming: Maximisation Problem

1. Solve the following linear programming problem using a simplex method. What are the optimal value of $R, X_1$ and $X_2$?

$$\max \ R = 10X_1 + 5X_2 \tag{95}$$

Subject to

$$25X_1 + 10X_2 \leq 1000 \tag{96}$$

$$20X_1 + 50X_2 \leq 1500 \tag{97}$$

where $X_1 \geq 0$ and $X_2 \geq 0$;

2. Write the dual of the above problem. Show that optimal solution of dual is equivalent to optimal solution of the primal problem.

3. Show that LP problem given above is a special case of non-linear problem.

Linear Programming: Simplex Algorithm for Maximisation

Table 6: Simplex Table 1

| | $R$ | $X_1$ | $X_2$ | $S_1$ | $S_2$ | Constant | Ratios |
|---|---|---|---|---|---|---|---|
| Row0 | 1 | -10 | -5 | 0 | 0 | 0 | |
| Row1 | 0 | 25 | 10 | 1 | 0 | 1000 | 40 |
| Row2 | 0 | 20 | 50 | 0 | 1 | 1500 | 75 |

Basic feasible solution

| $R$ | $X_1$ | $X_2$ | $S_1$ | $S_2$ | = | 0 | 0 | 0 | 1000 | 1500 |
|---|---|---|---|---|---|---|---|---|---|---|

Linear Programming: Simplex Algorithm for Maximisation

Table 7: Simplex Table 2

| | $R$ | $X_1$ | $X_2$ | $S_1$ | $S_2$ | Constant | Ratios |
|---|---|---|---|---|---|---|---|
| Row0 | 1 | 0 | -1 | 2/5 | 0 | 400 | |
| Row1 | 0 | 1 | 2/5 | 1/25 | 0 | 40 | 100 |
| Row2 | 0 | 0 | 42 | -4/5 | 1 | 700 | 16.7 |

Basic feasible solution

| $R$ | $X_1$ | $X_2$ | $S_1$ | $S_2$ | = | 400 | 40 | 0 | 0 | 16.7 |
|---|---|---|---|---|---|---|---|---|---|---|

Linear Programming: Simplex Algorithm for Maximisation

Basic feasible solution

| $R$ | $X_1$ | $X_2$ | $S_1$ | $S_2$ | = | 17500/42 | 700/21 | 700/42 | 0 | 0 |
|---|---|---|---|---|---|---|---|---|---|---|

Linear Programming: Duality

Every maximisation problem has corresponding minimisation problem. The revenue maximisation problem above has equivalent to the cost minimisation problem.

**Primal**

$$\max \ R = 10X_1 + 5X_2 \tag{98}$$

Table 8: Simplex Table 3

| | $R$ | $X_1$ | $X_2$ | $S_1$ | $S_2$ | Constant |
|---|---|---|---|---|---|---|
| Row0 | 1 | 0 | 0 | 8/21 | 1/42 | 17500/42 |
| Row1 | 0 | 1 | 0 | 1/25 | -1/105 | 700/21 |
| Row2 | 0 | 0 | 1 | -2/105 | 1/42 | 700/42 |

1. Subject to

$$\begin{bmatrix} 25 & 10 \\ 20 & 50 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \leq \begin{bmatrix} 1000 \\ 1500 \end{bmatrix}; \; X_1 \geq 0; X_2 \geq 0 \tag{99}$$

This is equivalent to minimising the cost

$$Min \; C = 1000Y_1 + 1500Y_2 \tag{100}$$

subject to:

$$\begin{bmatrix} 25 & 20 \\ 10 & 50 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \geq \begin{bmatrix} 10 \\ 5 \end{bmatrix}; \; Y_1 \geq 0; Y_2 \geq 0 \tag{101}$$

Linear Programming: Fundamental Theorems of Duality
Two fundamental theorems of duality:

1. Optimal values of the primal and the dual objective functions are always identical, provided that optimal feasible solution does exist.
2. If a certain choice variable in a linear programme is optimally nonzero then the corresponding dummy variable should be equal to zero. Similarly if a certain choice variable in a linear programme is optimally zero then the corresponding dummy variable in the linear programme should be non-zero.

**Homework: Market model and linear programming**

1. Write a GAMS code for the demand and supply model given above. Solve it using your code. Propose alternative scenarios.
2. Solve the linear programming problem using excel.

# 3 Macroeconomic Modelling and Programming

Growth and fluctuations are two major issues in macroeconomics.
Think of following issues on economic growth

1. What are the sources of current crisis?
2. Why the four-fifth of the World is still underdeveloped?
3. Why there is a North-South divide in per-capita income?

4. Story of productivity growth: impact of industrial to internet revolutions
5. Examination of poverty alleviation and economic growth
6. How much can human capital contribute towards economic growth?
7. Problems in transfer and adoption of technology
8. Why cannot all countries grow at the same rate?
9. What is the best technology to achieve higher rate of growth?
10. Balanced versus unbalanced growth
11. Does economic growth promote economic inequality?
12. Do higher environmental standards reduce the rate of growth?
13. Conflict, coalition and economic growth
14. Economic costs of conflicts and HIV in Africa
15. How much spending on research and development promote economic growth?
16. Infrastructure and economic growth

## 3.1 Growth models

Economic growth issue is often analysed summarising stylized growth facts and with the help of the neoclassical growth model as

Kaldor had summarised stylized facts of economic growth as:

- labour input has grown more slowly than capital and output. Therefore capital per capita (K/L) and output per capita (Y/L) have increased secularly.
- capital output ratio (K/Y) has remained fairly constant, has had no discernible trend and has more or less converged across industrialised economies.
- rate of return on capital (profit) and the real interest rate have no trend whereas real wage rates have followed a rising trend. Secular rises in productivity (Y/L) and per capita capital (K/L) have translated into higher real wages.
- share of income devoted to capital (r.K/Y) and to labour (w.L/Y) show no trend and remain fairly constant as the capital stock per person tends to grow along with output per person.

Neoclassical (Solow) growth model is given by following equations
Output

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} \tag{102}$$

where $\alpha + \beta = 1$
Saving

$$S_t = sY_t \tag{103}$$

Investment requirements

$$I_t = (n + \delta) K_t \tag{104}$$

Capital Accumulation

$$K_t = (1 - \delta) K_{t-1} + I_t \tag{105}$$

Market Clearing

$$Y_t = C_t + S_t = C_t + I_t; \qquad I_t = S_t \tag{106}$$

Percapita Output per Effective Worker

$$\tilde{y}_t = \frac{Y_t}{A_t L_t} = \frac{A_t K_t^{\alpha} L_t^{\beta}}{A_t L_t} = \frac{K_t^{\alpha}}{L_t^{1-\beta}} = k_t^{\alpha} \tag{107}$$

Fundamental Equation of Economic Growth

$$dk/dt = \dot{k} = s\tilde{y}_t - (n + \delta) k_t \tag{108}$$

There is no change in per capita capital in the steady state

$$\frac{dk/dt}{k_t} = \frac{\dot{k}}{k_t} = 0 \tag{109}$$

$\frac{dk/dt}{k_t} > 0$ before the steady state and $\frac{dk/dt}{k_t} < 0$ after the steady state.
Solution of the model for the steady state

$$dk/dt = \dot{k} = s\tilde{y}_t - (n + \delta) k_t \tag{110}$$

$$\frac{dk/dt}{k_t} = \frac{\dot{k}}{k_t} = sk_t^{\alpha-1} - (n + \delta) = 0 \tag{111}$$

$$sAk^{\alpha} = (n + \delta) k \tag{112}$$

Per capita capital in the steady state

$$k = \left( \frac{sA}{(n + \delta)} \right)^{\frac{1}{1-\alpha}} \tag{113}$$

Per capita output in the steady state

$$y = A \left( \frac{sA}{(n + \delta)} \right)^{\frac{\alpha}{1-\alpha}} \tag{114}$$

Percapita consumption in the steady state

$$c = (1-s)\,\tilde{y} = (1-s)\,A\left(\frac{sA}{(n+\delta)}\right)^{\frac{\alpha}{1-\alpha}} \tag{115}$$

Main theoretical results

- Countries with higher (lower) saving rate have higher (lower) steady state level of output .
- Countries with higher (lower) level of technology have higher (lower) level of output in the steady state.
- Countries with higher rate of population growth rate have lower level of output in the steady state.
- Countries with higher capital share have higher output in the steady state.
- Countries which differ in the initial capital stock eventually reach to the same output level in the steady state. Global economy converges in percapita income if all countries have the same technology.
- Growth of per capita income is determined by the rate of technical progress in the steady state; (saving rate only determines the level of output not its growth rate).

Late economists popularised endogenous growth models. Knowledge is very powerful in this model

Production Technology

$$Y_t = A_t K_t \tag{116}$$

Capital Accumulation

$$K_t = (1-\delta)\,K_{t-1} + I_t \tag{117}$$

Market Clearing

$$Y_t = C_t + S_t = C_t + I_t; \tag{118}$$

No diminishing returns to capital, marginal productivity of capital at time t is:

$$\frac{\partial Y_t}{\partial K_t} = A_t \tag{119}$$

Endogenous Growth

$$\ln(Y_t) = \ln(A_t) + \ln(K_t) \tag{120}$$

Growth rate of output (by log differentiation w.r.t. time)

$$g_y = g_A + g_K \tag{121}$$

Growth rate of output = Growth rate of TFP+growth rate of capital

Higher rate of saving implies more investment.

$$I_t = S_t \tag{122}$$

$$g_K = \frac{\partial K/\partial t}{K} = \frac{sY}{K} \tag{123}$$

That means larger capital stock $[K_t = (1-\delta) K_{t-1} + I_t]$ and higher level of output $[Y_t = A_t K_t.]$.Efficiency of the financial system is important for capital formation.

Human capital augmented Solow model.

$$Y_t = A_t K_t^{\alpha} L_t^{\beta} H^{\gamma} \tag{124}$$

where $\alpha + \beta + \gamma = 1$ and $A_t$ is an index of Hicks neutral technical knowledge.

A certain country with parameter values $\alpha = 0.3,\ \beta = 0.5$ and $\gamma = 0.2$ had output $(Y_t)$ growing by 6 percent, capital $(K_t)$ by 3 percent, labour $(L_t)$ by 2 percent and human capital $(H_t)$ by 2 percent. What was the growth rate of technical progress $(g_A)$ in this country?

Data on economic growth can be collected from various sources. Summers and Heston Dataset (Penn World Tables http://pwt.econ.upenn.edu/ ; https://www.aeaweb.org/rfe/) is the most comprehensive one.

**GAMS Code for Solow Model** $Title Solow Model

```
set t time /t1*t10/
tfirst
tlast;
tfirst(t) = Yes$(ord(t) eq 1);
tlast(t) = Yes$(ord(t) eq card(t));
alias (t, tt);
PARAMETERS
a capital share
b labour share
d depreciation rate
n population growth rate
s savings rate
L0 initial population
K0 initial capital stock
L(t) lablour
;
a=0.4;
b = 1-a;
d=0.03;
n=0.02;
s=0.2;
l0=100;
k0=100;
L(t) =L0$tfirst(t)+((1+n)**ORD(T)*L0)$(ord(t) ne 1);
Variables
```

```
Y(t) output
Sv(t) savings
I(t) investment
C(t) consumption
DP(t) depreciation
K(t) capital stock
CT total consumption;
Equations
output(t) output
saving(t) saving
invest(t) investment
consump(t) consumption
tcons total consumption
depr(t) depreciation
capital(t) capital stock
;
output(t)..
y(t) =e= (k(t)**A)*L(t)**b;
saving(t)..
Sv(t) =e=s*Y(t);
invest(t)..
I(t) =e= (n+d)*K(t);
consump(t)..
C(t) =e=Y(t)-Sv(t);
depr(t)..
DP(t) =e= d*K(t);
capital(t)..
K(t) =e= K0$tfirst(t)+((1-d)*K(t-1)+Sv(t-1)-I(t-1))$(ord(t) ne 1);
tcons..
ct =e=sum(t,c(t));
Model solow/output,saving,invest,consump,tcons,depr,capital/;
solve solow maximising CT using nlp;
Parameter report;
report(t,"capital") = K.L(t);
report(t,"LABOUR") = L(T);
report(t,"output") = Y.L(t);
report(t,"SAVING") = SV.L(t);
report(t,"DEPR") = DP.L(t);
report(t,"INVEST") = I.L(t);
report(t,"CONSUMP") = C.L(t);
report(t,"NET INV") = Sv.L(t)-I.L(t);
DISPLAY REPORT;
```

**Homework: Economic growth**

1. Compute steady state in the Solow growth model in excel
2. Construct growth data set from Summer's and Heston dataset. http://pwt.econ.upenn.edu/
3. Estimate one growth regression and study the significance of coefficients.

## 3.2 Macro models of fluctuations

1. What determines the level of GDP, employment, consumption, investment, exports and imports in the short run?
2. Why some countries have more deficit than others?
3. Why are Keynesian models applicable more in some countries than in others?
4. Why the rates of unemployment are higher in rigid labour markets?
5. Examination economic problems when savings are not equal to investment
6. Should households save more to make economy grow faster?
7. Resource imbalances and economic crises
8. First, second and third theories of economic crises.
9. What are the best policy rules for stability and growth?
10. Trade-off between unemployment and inflation?
11. Can independent central banks do better than government controlled ones?
12. How can exchange rate instability be harmful for an economy?
13. Credibility of public policy and market reactions

These issues require business cycle models

### 3.2.1 Keynesian model

A simple Keynesian model of the real sector

$$Y = C + I + G \tag{125}$$

$$C = a + b(Y - T); \quad a > 0 \ , \ 0 < b < 1 \tag{126}$$

Rearrange for a matrix

$$Y - C = I + G \tag{127}$$

$$-bY + C = a - bT \tag{128}$$

$$\begin{bmatrix} 1 & -1 \\ -b & 1 \end{bmatrix} \begin{pmatrix} Y \\ C \end{pmatrix} = \begin{pmatrix} I+G \\ a-bT \end{pmatrix} \tag{129}$$

or

$$\begin{pmatrix} Y \\ C \end{pmatrix} = \begin{bmatrix} 1 & -1 \\ -b & 1 \end{bmatrix}^{-1} \begin{pmatrix} I+G \\ a-bT \end{pmatrix} \tag{130}$$

Using Cramer's rule

$$Y = \frac{\begin{vmatrix} I+G & -1 \\ a-bT & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -b & 1 \end{vmatrix}} = \frac{(I+G)+(a-bT)}{1-b} \tag{131}$$

$$C = \frac{\begin{vmatrix} 1 & I+G \\ -b & a-bT \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ -b & 1 \end{vmatrix}} = \frac{(a-bT)+(I+G)}{1-b} \tag{132}$$

### 3.2.2 Homework: Keynes-Hicks ISLM Model

Find the equilibrium level of output and interest in the Keynes-Hicks Macroeconomic model as given by following equations.

Consumption function

$$\mathbf{C}_t = \beta_0 + \beta_1 (Y_t - T_t) \tag{133}$$

Investment

$$\mathbf{I_t} = \mu_0 - \mu_1 R_t \tag{134}$$

Taxes

$$\mathbf{T_t} = T_0 + t_1 Y_t \tag{135}$$

Imports

$$M_{\mathbf{t}} = m_0 + m_1 Y_t \tag{136}$$

Macro balance

$$Y_t = C_t + I_t + G_t + X_t - M_t = C_t + T_t + S_t \tag{137}$$

$$\left(\frac{\overline{MM}}{P}\right)_t = b_0 + b_1 Y_t - b_2 R_t \tag{138}$$

### 3.2.3 Samuelsonian Multiplier Accelerator Model

Macro balance

$$Y_t = C_t + I_t + G_0 \tag{139}$$

Consumption function

$$C_t = \gamma Y_{t-1}; \qquad 0 < \gamma < 1 \tag{140}$$

Investment

$$I_t = \alpha \left( C_t - C_{t-1} \right); \qquad \alpha > 1 \tag{141}$$

Equilibrium (putting $C_t$ and $I_t$ in $Y_t$ ): second order difference equation

$$Y_t = \gamma \left( 1 + \alpha \right) Y_{t-1} - \gamma \alpha Y_{t-2} + G_0 \tag{142}$$

Income is constant in the steady state

$$Y_t = Y_{t-1} = Y_{t-2} = \overline{Y} \tag{143}$$

Business Cycle in Samuelsonian Multiplier Accelerator Model

$$\overline{Y} - \gamma \left( 1 + \alpha \right) \overline{Y} + \gamma \alpha \overline{Y} = G_0 \tag{144}$$

Steady state output

$$\overline{Y} = \frac{G_0}{1 - \gamma \left( 1 + \alpha \right) + \gamma \alpha} = \frac{G_0}{1 - \gamma} \tag{145}$$

Transitional dynamics depends on the homogenous part of this second order difference equation

$$Y_t - \gamma \left( 1 + \alpha \right) Y_{t-1} + \gamma \alpha Y_{t-2} = 0 \tag{146}$$

Solution of the Samuelsonian Multiplier Accelerator Model
Transitional dynamics (replace $Y_t = Ab^t$ in homogenous equation).

$$\begin{aligned} Y_t - \gamma \left( 1 + \alpha \right) Y_{t-1} + \gamma \alpha Y_{t-2} &= 0 \\ Ab^t - \gamma \left( 1 + \alpha \right) Ab^{t-1} + \gamma \alpha Ab^{t-2} &= 0 \end{aligned} \tag{147}$$

$$b^2 - \gamma \left( 1 + \alpha \right) b + \gamma \alpha = 0 \tag{148}$$

Cycle depends on roots of the quadratic equation

$$b_1, b_2 = \frac{\gamma \left( 1 + \alpha \right) \pm \sqrt{\gamma^2 \left( 1 + \alpha \right)^2 - 4 \gamma \boldsymbol{\alpha}}}{2} \tag{149}$$

Three Cases in Samuelsonian Multiplier Accelerator Model
Distinct real root case (no cycle)

$$\gamma^2 (1+\alpha)^2 > 4\gamma\alpha \tag{150}$$

Repeated real root case (no cycle)

$$\gamma^2 (1+\alpha)^2 = 4\gamma\alpha \tag{151}$$

Complex root case (cycle)

$$\gamma^2 (1+\alpha)^2 < 4\gamma\alpha \tag{152}$$

Complete solution

$$Y_t = A_1 b_1^t + A_2 b_2^t + \overline{Y} \tag{153}$$

$Y_t = A_1 R^t (\cos\theta \cdot t + i \cdot \sin\theta \cdot t) + A_2 R^t (\cos\theta \cdot t - i \cdot \sin\theta \cdot t) + \overline{Y}$
**Open economy macro model**
An Example of Mundell-Fleming Open Economy Macroeconomic Model
National income

$$Y = C(Y-T) + I(Y, i-\pi^e) + G + NX(Y, Y^f, \frac{eP^*}{P}) \tag{154}$$

Money market

$$\left(\frac{M}{P}\right) = L(Y, i) \tag{155}$$

Interest rate parity

$$i = r + \pi^e \tag{156}$$

Real exchange rate

$$\varepsilon = \frac{eP^*}{P} \tag{157}$$

An Example of Mundell-Fleming Open Economy Macroeconomic Model
Balance of payment

$$NX = KF(r - r^*) \tag{158}$$

Aggregate supply

$$Y = \overline{Y} + \alpha(P - P^e) \tag{159}$$

Natural rate of output

$$\overline{Y} = F\left(\overline{K}, \overline{L}\right) \tag{160}$$

- Endogenous variables: $Y,\ i,$ **r**. **P**,$e,\ \varepsilon,\ \overline{Y}$

- Exogenous policy variables: T, $G$, $M, P^e, \pi^e, r^*, P^*, \overline{K}, \overline{L}, Y^f$
- Macroeconometric model can be estimated by PcGive. See Keynesian.xls and samuel.xls.

### 3.2.4 Homework: construct macroeconomic data

Construct a data set on macro economy of your choice.

Constructing Data for Analysis: Step by Step Guidelines

Connect to http://www.esds.ac.uk/international/ Choose direct links to macro data

Important Steps for extracting data

I. World Bank Data (World Bank data Indicator

1. Click on Direct Links to Macro Data
2. Choose World Bank Data
3. Select University of Hull
4. Put Shibboleth user name and pass word;
5. Complete the registration process required by data
6. Select World Bank Development Indicators
7. Select Year 1960 -2014 (all can be selected by a tick mark)
8. Select a country (e.g. China, India, UK, US, South Africa, Brazil, South Korea)
9. Select a series (e. g. Population between 15-64; and population growth rate) ; can search for population
10. Click on show Table
11. Adjust row and column dimensions of the table by moving around icons
12. Download data in *.CSV format
13. Open the data file just created
14. Make some time series graph
15. Next time; add few more variables like DGP per capita constant 2000 dollars; Gross fixed capital formation % of GDP; Final consumption Expenditure as a % GDP; Current account balance as a % of GDP; General Government final consumption % of GDP; GDP constant 2000 $

II. IMF World Economic Outlook (WEO) data

1. Steps 1 -5 as above
2. Select IMF WEO data
3. Select World Economic Outlook
4. Select Euro Aria
5. Select crude oil price, output gap, unemployment rate, inflation average consumer price)
6. Select all years 1991-2010
7. Show Table; Download the data; Open and Excel.
8. Do macro analysis.

Look at world economic outlook dataset in www.imf.org.

III. Eurostat New Cornos (Data for EU countries)

1. Steps 1 -5 as above
2. Select Eurostat New Cornos
3. Select Economy and Finance
4. Eurostat
5. Exchange rates
6. Nominal effective exchange rates

7. Real effective exchange rates

IV DATASTEAM

This is extremely rich real time data set on companies listed in stock markets. It also contains data of most macroeconomic variables. Get the Password from the Masters Office or Research Methods module eBridge. It is now available in DW-CS1 (access code for the room CB2194). Datastream is accessible from all machines there.

Get acquainted with this very useful dataset for financial and economic analysis (see indices, stock prices, EPS, financial ratios, debt and ownership structures, capital structure, revenue, profits, balance sheets, annual reports, growth ratios). Bloomberg database is also available in this room.

# 4 General Equilibrium Trade Model

**Two Country Ricardian Trade Model**

- There are two countries indexed by $j$, producing two goods, manufacturing and services.
- Each of them have an option to be self reliant or to trade on the basis of comparative advantage.
- Under the ISI regimes countries favoured to be self reliant and infant industries were protected by tariffs and non-tariff barriers. After numerous rounds of trade negotiations under GATT/WTO over the years, all countries now have realised that the autarky solutions like this are economically inefficient. In contrast
- trade is mutually beneficial for trading nations and raises welfare in both countries. Aim of this section is to illustrate on these statements analytically and numerically with a small and transparent example.
- For this it is assumed that each country $j$ specialises in commodities that it is more efficient and engages in trade.
- The exchange rate is determined by the relative prices of two commodities in the global market.

Two Country Ricardian Trade Model

- Preferences in country $j$ are expressed by its utility function in consumption of good 1 and 2 , $C_1^j$ and $C_2^j$ respectively:

$$max \quad U^j = \left(C_1^j\right)^{\alpha^j} \left(C_2^j\right)^{1-\alpha^j} \tag{161}$$

Income of country $j$ is obtained from the wage income in sector 1 and sector 2 plus the transfers to country $j$

$$I^j = w_1^j L_1^j + w_2^j L_2^j + TR^j \tag{162}$$

where $L_1^j$ and $L_2^j$ are labour employed in sector 1 and sector 2 $w_1^j$ and $w_2^j$ are corresponding wages respectively and $TR^j$ is the transfer income.

Technology constraints in sector 1 in country $j$

$$X_1^j = a_1^j.L_1^j \tag{163}$$

Two Country Ricardian Trade Model
where $a_1^j$ is the productivity of labour in sector 1 in country $j$.
Technology constraints in sector 2 in country $j$

$$X_2^j = a_2^j.L_2^j \tag{164}$$

where $a_2^j$ is the productivity of labour in sector 2 in country $j$.
Resource constraint in country $j$ defined by the labour endowment as:

$$L^j = L_1^j + L_2^j \tag{165}$$

Production possibility frontier of country $j$ now can be defined as

$$L^j = \frac{1}{a_1^j}.X_1^j + \frac{1}{a_2^j}.X_2^j \tag{166}$$

Two Country Ricardian Trade Model
Given above preferences the demand for good 1 in country $j$ is

$$C_1^j = \frac{\alpha^j.I^j}{P_1} \tag{167}$$

the demand for good 2 therefore is:

$$C_2^j = \frac{\left(1-\alpha^j\right).I^j}{P_2} \tag{168}$$

Global market clearing for good 1

$$\sum_j^N C_{1,j} = \sum_j^N X_{1,j} \tag{169}$$

Global market clearing for good 2

$$\sum_j^N C_{2,j} = \sum_j^N X_{2,j} \tag{170}$$

Capital Accumulation in Overlapping Generation Model

- Theoretically two trade arrangements are possible in this model. First one is an autarky equilibrium in which each country is separate and isolated from another. It produces just for its own consumption and no trade take place between these two countries.
- Such autarky solution is close to the production arrangement when countries were adopting ISI trade strategy.

**Proposition 1**

- *Autarky solution is Pareto dominated by trade equilibrium for reasonable parameters of preferences and technology.*
- This is proven below by analytical and numerical solutions.

#### 4.0.5 Analytical solutions of autarky and specialisation

A Lagrangian function is used to express how each country $j$ maximises welfare subject to its production possibility frontier constraint under the autarky equilibrium as:

$$\pounds_j = X_{1,j}^{\alpha_j} X_{2,j}^{(1-\alpha_j)} + \lambda \left[ L_j - \frac{1}{a_1^j} X_{1,j} - \frac{1}{a_2^j} X_{2,j} \right] \tag{171}$$

First order conditions with respect to $X_1^j$ and $X_2^j$ and $\lambda$ as:

$$\frac{\partial \pounds_j}{\partial X_{1,j}} = \alpha_j X_{1,j}^{\alpha_j - 1} X_{2,j}^{(1-\alpha_j)} - \frac{\lambda}{a_1^j} = 0 \tag{172}$$

$$\frac{\partial \pounds_j}{\partial X_{2,j}} = (1-\alpha_j) X_{1,j}^{\alpha_j} X_{2,j}^{(-\alpha_j)} - \frac{\lambda}{a_2^j} = 0 \tag{173}$$

Analytical solutions of autarky and specialisation

$$\frac{\partial \pounds_j}{\partial \lambda} = L_j - \frac{1}{a_1^j} X_{1,j} - \frac{1}{a_2^j} X_{2,j} = 0 \tag{174}$$

From the first two first order conditions $\frac{\alpha_j X_{1,j}^{\alpha_j - 1} X_{2,j}^{(1-\alpha_j)}}{(1-\alpha_j) X_{1,j}^{\alpha_j} X_{2,j}^{(-\alpha_j)}} = \frac{\alpha_j}{(1-\alpha_j)} \frac{X_{2,j}}{X_{1,j}} = \frac{a_2^j}{a_1^j}$

$$X_{2,j} = \frac{(1-\alpha_j)}{\alpha_j} \frac{a_2^j}{a_1^j} X_{1,j} \tag{175}$$

optimal value of $X_{1,j}$ is found now putting this condition in the production possibility frontier constraint.

$$\frac{1}{a_1^j} X_{1,j} + \frac{1}{a_2^j} X_{2,j} = \frac{1}{a_1^j} X_{1,j} + \frac{1}{a_2^j} \frac{(1-\alpha_j)}{\alpha_j} \frac{a_2^j}{a_1^j} X_{1,j} = \frac{1}{a_1^j} X_{1,j} \left[ 1 + \frac{(1-\alpha_j)}{\alpha_j} \right] = L_j \tag{176}$$

Analytical solutions of autarky and specialisation

$$X_{1,j} = \alpha_j a_1^j L_j \tag{177}$$

Similarly the optimal value of $X_{2,j}$ is found by

$$X_{2,j} = \frac{(1-\alpha_j)}{\alpha_j} \frac{a_2^j}{a_1^j} X_{1,j} = \frac{(1-\alpha_j)}{\alpha_j} \frac{a_2^j}{a_1^j} \alpha_j a_1^j L_j = (1-\alpha_j) a_2^j L_j \tag{178}$$

For each of $j$ country amount produced depends on productivity and preferences parameters and the endowment of its labour input. The autarky welfare level is:

$$U^j = (X_{1,j})^{\alpha^j} (X_{2,j})^{1-\alpha^j} = \left( \alpha_j a_1^j L_j \right)^{\alpha_j} \left( (1-\alpha_j) a_2^j L_j \right)^{(1-\alpha_j)} \tag{179}$$

Summary of two country trade model

- Thus the level of welfare in country $j$ is determined in terms of its preferences for consumption of good 1 and 2 as reflected by $\alpha_j$ and its own production technology as reflected in $a_1^j$ and $a_2^j$.
- Numerical version of this model is applied to China and the US taking the population as rough indicator of its resource in production. US has 365 million population and China has 1200 million population. US is more productive in producing services goods $X_2$ whereas China has more advantage in producing manufacturing goods $X_1$. Preferences are similar but technologies are different. These parameters are set out in Table 1.

Table 9: Parameters of the Autarky Model

| | $\alpha$ | $a_1$ | $a_2$ | $L$ |
|---|---|---|---|---|
| US | 0.6 | 2 | 5 | 365 |
| China | 0.6 | 5 | 2 | 1200 |

Summary of two country trade model

- Under the autarky equilibrium these two economies are completely isolated and produce only for domestic consumption. The optimal production and consumption and employment of labour for both sectors, prices of commodities and labour, and utility for the representative household are as given in Table 2. In per capita terms citizens of the US and China have welfare of 1.46 and 1.76 respectively.

Table 10: Parameters of the Autarky Model

| | $X_1$ | $X_2$ | $L_1$ | $L_2$ | $U$ | $p_2$ |
|---|---|---|---|---|---|---|
| US | 438 | 730 | 219 | 146 | 535.8 | 2.5 |
| China | 3600 | 960 | 720 | 480 | 2121.7 | 0.27 |

Each country produces both goods in no trade equilibrium which as explained here is very inefficient. Welfare can be improved by making these countries trade.

Analytical solutions for trade equilibrium

A representative household in each country maximises its welfare subject to its budget constraint.

Demand for goods are derived by standard constrained optimisation on supply side for each country $j$ . Under trade equilibrium it is optimal for each country to specialise in goods in which it has comparative advantage. The optimisation problem and the first order conditions for constrained optimisation are given as follows:

$$\pounds_j = X_{1,j}^{\alpha_j} X_{2,j}^{(1-\alpha_j)} + \lambda \left[ I_j - P_1 X_{1,j} - P_2 X_{2,j} \right] \tag{180}$$

First order conditions:

$$\frac{\partial \pounds_j}{\partial X_{1,j}} = \alpha_j X_{1,j}^{\alpha_j - 1} X_{2,j}^{(1-\alpha_j)} - \lambda P_1 = 0 \tag{181}$$

$$\frac{\partial \pounds_j}{\partial X_{2,j}} = (1-\alpha_j) X_{1,j}^{\alpha_j} X_{2,j}^{(-\alpha_j)} - \lambda P_2 = 0 \tag{182}$$

Analytical solutions for trade equilibrium

$$\frac{\partial \pounds_j}{\partial \lambda} = I_j - P_1 X_{1,j} - P_2 X_{2,j} = 0 \tag{183}$$

$$\frac{\alpha_j X_{1,j}^{\alpha_j - 1} X_{2,j}^{(1-\alpha_j)}}{(1-\alpha_j) X_{1,j}^{\alpha_j} X_{2,j}^{(-\alpha_j)}} = \frac{\alpha_j}{(1-\alpha_j)} \frac{X_{2,j}}{X_{1,j}} = \frac{P_1}{P_2} \tag{184}$$

$$X_{2,j} = \frac{(1-\alpha_j)}{\alpha_j} \frac{P_1}{P_2} X_{1,j} \tag{185}$$

$P_1 X_{1,j} + P_2 X_{2,j} = P_1 X_{1,j} + P_2 \frac{(1-\alpha_j)}{\alpha_j} \frac{P_1}{P_2} X_{1,j} = I_j$

$$X_{1,j} = \frac{\alpha_j I_j}{P_1};\ X_{2,j} = \frac{(1-\alpha_j) I_j}{P_2} \tag{186}$$

Analytical solutions for trade equilibrium

Global market clearing conditions for goods 1 and 2 are

$$\sum_j^N X_{1,j} = X_1 \tag{187}$$

$$\sum_j^N X_{2,j} = X_2 \tag{188}$$

Prices adjust until this equilibrium condition holds.

Under complete specialisation, country 1 US specialises in services $X_2$ and produces 1825 units of it. China specialises in manufacturing $X_1$ goods and produced 6000 units of it. It is easy to determine China's income if we choose good 1 as numeraire setting $P_1 = 1$.

Analytical solutions for trade equilibrium

$$I^c = P_1 X_1 = 1 \times 6000 = 6000 \tag{189}$$

Relative price of good 2, $P_2$ need to be determined to find the level of income in the US. This can be done using the global market clearing condition

$$\frac{\alpha^u . I^u}{P_1} + \frac{\alpha^c . I^c}{P_1} = 0.6\,(1825 \times P_2) + 0.6\,(6000) = 6000 \tag{190}$$

$$P_2 = \frac{6000 - 3600}{1095} = 2.192 \tag{191}$$

Now it is easy to determine the income of the US as:

$$I^u = P_2 X_2 = 365 \times 5 \times P_2 = 1825 \times P_2 = 1825 \times 2.192 = 4000.4 \tag{192}$$

Analytical solutions for trade equilibrium

Since income level for both China and the US are determined, it is now easy to determine the level of demand in both countries:

$$X_{1,u} = \frac{\alpha_u I_u}{P_1} = 0.6\,(4000.4) = 2401.6;\ X_{1,ch} = \frac{\alpha_{ch} I_{ch}}{P_1} = 0.6\,(6000) = 3600 \tag{193}$$

$$X_{2,u} = \frac{(1-\alpha_u)\,I_u}{P_2} = \frac{0.4\,(4000.4)}{2.192} = 730; X_{2,ch} = \frac{(1-\alpha_{ch})\,I_{ch}}{P_2} = \frac{0.6\,(6000)}{2.192} = 1462.3 \tag{194}$$

Solutions of both autarky and trade equilibria are given in Table 3 and 4. Given the preferences and technology specifications, with complete specialisation both countries gain from trade. Comparative static analysis of trade can be done changing the preference or technology parameters.

Analytical solutions for trade equilibrium

Table 11: Comparing Specialisation and Autarky Regimes

| | Production | | | | Consumption | | | |
|---|---|---|---|---|---|---|---|---|
| | Autarky | | Trade | | Autarky | | Trade | |
| | $X_1$ | $X_2$ | $X_1$ | $X_2$ | $C_1$ | $C_2$ | $C_1$ | $C_2$ |
| US | 438 | 730 | 0 | 1825 | 438 | 730 | 1600.2 | 730 |
| China | 3600 | 960 | 6000 | 0 | 3600 | 960 | 3600 | 1642.3 |

Analytical solutions for trade equilibrium

Table 12: Comparing Employment and Welfere under Specialisation and Autarky

| Employment | | | | Uitlity | |
|---|---|---|---|---|---|
| Autarky | | Trade | | Autarky | Trade |
| $L_1$ | $L_2$ | $L_1$ | $L_2$ | $U$ | $U$ |
| 219 | 146 | 0 | 365 | 535.8 | 1169.1 |
| 720 | 480 | 1200 | 0 | 2121.7 | 2248 |

Gains from trade may be distributed differently across countries (Bhattarai and Whalley (2006)). Further there are opportunities for bargaining on the share of those gains particularly from dynamic strategic considerations and the basic elements required for such dynamic model is provided in the next section.

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**Homework: Estimating a macroeconomic model** Estimate a simple macroeconomic model using PcGive and use this for forecasting revenues and budget deficit given the target of the public expenditure. Empirical Part: Exercise in PcGive

- construct data set in macroeconomic variables ( Y, C, I , G, T , X, M, MS, i, inflation, wage rate, exchange rate etc)
- save data in *.csv format
- Start GiveWin and PcGive and open data file
- choose multiple equation dynamic modelling
- determine endogenous and exogenous variables and run simultaneous equation using 3SLS or FIML
- Study coefficients
- Change policy variables and construct few scenarios

# 5 Statistical Analysis

Central tendency

Statistics is about summarising data on discrete or random variables using descriptive statistics and trying to find out their probability distributions. Main issues are

Mean or average (measures central tendency of the data):

$$\overline{X} = \frac{\sum X}{N} \tag{195}$$

Variance and standard deviations measure dispersion of the data:

$$var\left(X\right) = \frac{\sum \left(X - \overline{X}\right)^2}{N-1} \tag{196}$$

$$SD\left(X\right) = \widehat{\sigma} = \sqrt{var\left(X\right)} = \sqrt{\frac{\sum \left(X - \overline{X}\right)^2}{N-1}} \tag{197}$$

Coefficient of variation

$$CV = \frac{\widehat{\sigma}}{\overline{X}} \times 100 \tag{198}$$

Correlation coefficient measures the association between two variables (it does not imply causality):

$$\rho_{x,y} = \frac{\sum \left(X - \overline{X}\right) \sum \left(Y - \overline{Y}\right)}{\sqrt{var\left(X\right)}\sqrt{var\left(Y\right)}}; \quad 0 \leq \rho \leq 1 \tag{199}$$

#### 5.0.6 Normal distribution and its family

Density function (data generating process) of a normally distributed variable $y$ is given by

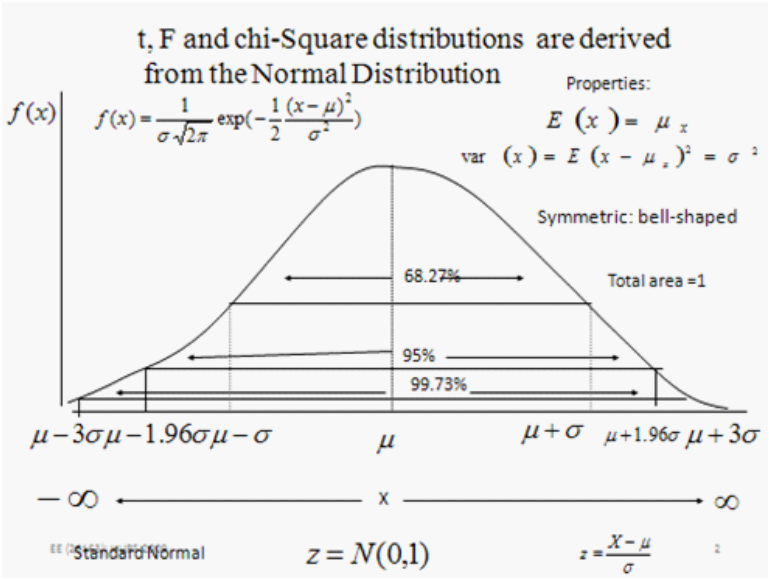
$$f(y) = \frac{1}{\sqrt{2\pi\sigma_y^2}} \exp\left[-\frac{1}{2}\frac{\left(y-\mu\right)^2}{\sigma_y^2}\right] \tag{200}$$

Normal distribution is symmetric around the mean.

Table 13: Percentage area under the Normal curve by standard deviation

| | $\sigma$ | $1.96\sigma$ | $3\sigma$ |
|---|---|---|---|
| Area | 68.27% | 95% | 99.73% |

Any normally distributed variable can be transformed into a standard normal variable with mean zero and variance of 1, $z = \frac{\overline{X}-\mu}{\sigma} \sim N\left(0,1\right)$. Normality is many circumstances desired property of a variable; many statistical tests are conducted to determine whether a variable is normally distributed or not. $t$, $F$ and $\chi^2$ distributions are derived from the normal distribution.




t-values ( for normally distributed x variable)

$$t = \frac{\overline{X} - \mu}{\widehat{\sigma}/\sqrt{N}}; \quad X \sim N(0,1) \tag{201}$$

Theoretical values of t are given in a t Table. Column of t-table have level of significance ($\alpha$) and rows have degrees of freedom.

Here $t_{\alpha,df}$ is t-table value for degrees of freedom ($df = n - k$) and $\alpha$ level of significance. df = 6-2=4.

Table 14: Relevant t-values (one tail) fron t-Table

| $(n, \alpha)$ | 0.05 | 0.025 | 0.005 |
|---|---|---|---|
| 1 | 6.314 | 12.706 | 63.657 |
| 2 | 2.920 | 4.303 | 9.925 |
| 4 | 2.132 | 2.776 | 4.604 |

$\chi^2$ distribution is the square of a normally distributed variable

$$\chi^2_k = \sum_k X^2_k \text{ for } \quad X \sim N(0,1) \tag{202}$$

F- statistics is the ratio of two $\chi^2$ distributed variables. It is widely used to determine whether variances of two variables significantly differ from each other.

F-value is the ratio of sum of squared normally distributed variables ($\chi^2$ ) adjusted for relevant degrees of freedom.

$$F = \frac{V_1/n_1}{V_2/n_2} = F(n_1, n_2) \tag{203}$$

Where $V_1$ and $V_2$ are variances of numerator and denominator and $n_1$and $n_2$ are degrees of freedom of numerator and denominator.

$H_0$: Variance are the same; $H_A$: Variance are different. $F_{crit}$ values are obtained from F-distribution table. Accept it if $F_{Calc} < F_{crit}$ and reject if $F_{Calc} > F_{crit}$ .

F- is ratio of two $\chi^2$ distributed variables with degrees of freedom $n2$ and $n1$.

Table 15: Relevant F-values from the F-Table

| | 1% level of significance | | | 5% level of significance | | |
|---|---|---|---|---|---|---|
| $(n2, n1)$ | 1 | 2 | 3 | 1 | 2 | 3 |
| 1 | 4042 | 4999.5 | 5403 | 161.4 | 199.5 | 215.7 |
| 2 | 98.50 | 99.00 | 99.17 | 18.51 | 19.00 | 19.16 |
| 4 | 21.20 | 18.00 | 16.69 | 7.71 | 6.94 | 6.59 |

$n1$ = degrees of freedom of numerator; $n2$ =degrees of freedom of denominator.

## 5.1 Issues in statistical analysis

A number of issues arise in statistical analysis. These can be summarised in following groups:

1. Data Structure:
   - Univariate representation - histograms, frequency- stem and leaf
   - Bivariate - box plots
   - Multivariate: graphical representation
   - Data transformation (logs, differences, multiplications, exponentiation)
2. Underlying assumption
   - Normality - mean, skewness and Kurtosis tests
   - Homoscedasticity
   - Linearty
   - No autocorrelation in errors
3. Problems of missing observations or outliers
   - Extent and randomness of missing data
   - Missing at random
   - Missing completely at random
   - Imputation methods
4. Problems of outliers
   - Univariate detection
   - Bivariate detection
   - Multivariate detection.

### 5.1.1 Homework: statistical analysis

Read data in score.xls and answer the following questions.

1. (a) Represent the data on scores in exams and earning using marginal and cumulative frequency diagrams.
   (b) What are means and variances of scores in exam 1 and exam 2? What are the coefficients of variation of scores in exams1 and exam 2?
   (c) What is the covariance of marks in exams 1 and 2?
   (d) What is the correlation coefficient of scores between exam 1 and 2?
   (e) If exam 1 weighs 100 percent but the scores in exam 2 weigh only 10 percent what would be the weighted aggregate mean score in these two exams? What would be the variance of weighted scores?
   (f) Exam 1 took place before exam 2. Test whether scores in exam 1 can predict scores in exam 2?
   (g) Predict scores in exam 2 for students who scored 6 and 8 in exam 1.
   (h) Test hypothesis whether scores in exam 1and exam 2 are significant determinants of earning. Why may earnings be negatively related with their score in the exams for full times students?
   (i) How can behaviors of teachers and students change the distribution of marks?
   (j) If the true mean was 6 for score 1 and 58 for score 2 find whether the current sample reflects the population using t-test.
   (k) Derive the standard normal distribution for score 2 and construct a 99 percent confidence interval for it.

## 5.2 Multivariate analysis

It is a branch of statistics. It aims to generate knowledge by processing quantitative and qualitative data using standard statistical techniques. It is about finding patterns in vast amount of information contained in surveys, statistical reports.

Mainly devided between dependent and independent analysis (see Hair JF, WC Black, BJ Babin, RE Anderson and RL Tatham (2006) ).

1. Dependent Analysis:
   - Multiple regression - Metric Variables (see econometrics section)
   - Time series analysis (see time series section)
   - Conjoint analysis - categorical dependent variables
   - Discriminant analysis - categorical dependent variables
   - Multivariate analysis of variance (MANOVA)
   - Canonical correlations
   - Structural equation modelling

2. Interdependent Analysis

   - Factor analysis
   - Cluster analysis
   - Multidimensional scaling
   - Correspondence analysis.

### 5.2.1 Conjoint analysis - categorical dependent variables

Evaluating how customers develop preferences for any type of object

Utility based on

Attributes of the product (colour, taste, content)

Overall utility by factors, level and treatment elements (solid, liquid; Brand )

Conjoint task

Determining the total worth of the product

Total worth of the product i ,j ni,j

= Part worth of level of factor 1+ Part worth of level of level j factor2 + Part worth of level of level j factor m

Utility, factors, level, stimuli

Goodness of fit measures: Pearson, Kendal's tau for estimation and validation samples; Part worth of estimates, rescaling and reversals, validation of results

### 5.2.2 Discriminant analysis - categorical dependent variables

Statistical technique to compare the means of a set of independent variables for two or more groups. Males differ from female, North- South, skilled vs unskilled.

Categorical dependent variable

Logistic regression

Frequency distributions for two/multiple of groups.

Steps for modelling discriminant analysis

Sort observations by groups and check the differences in group means

Determine independent variables and sample size

Assumptions - normality, linearity, no multicollinearity, equal dispersions

Estimation of the characteristic function - simultaneous or stepwise

Check the significance of the discriminant function

Assess prediction accuracy

Interpret results - discriminant weights, loadings, partial fractions, split samples and cross validation

### 5.2.3 Multivariate analysis of variance (MANOVA)

Assessing group differences

t-test on differences of mean for two variables

ANOVA

Total variance

Within group variances
Between group variance
F test; level of significance and critical values
Multivariate analyses of variance (MANOVA)
Hotellings $T^2$ test

$$T^2 = \frac{p\,(N_1 + N_1 - 1)}{N_1 + N_1 - p - 1} \times F_{crit} \tag{204}$$

Multivariate analysis of covariance.

### 5.2.4 Canonical correlations

Theory developed by Hotelling (1936, 1936)
Correlating several metric dependent variables and metric independent variables simultaneously;
Identifying a subset of variables with the largest correlation from the set of many variables;
Finding a linear combination among those variables that maximises correlation between them
Do measurement of skulls and intelligence score elate to each other?
Investment and profit?
Application in VAR model - cointegration analysis
Business example
Survey with 50 questions for a world class company and a particular company,
Do correlations exists between the particular company and the world class company.

### 5.2.5 Structural equation modelling

Explaining relationships among multiple variables; finding interrelationships
Foundation on factor analysis and multiple regression analysis
Constructs (exogenous variables) and latent factors
Analysis of the covariance structure
Model to define the entire set of relations
Path diagrams
Avoid spurious relations
Simultaneous maximum likelihood estimation

### 5.2.6 Factor analysis

Condensing the underlying structure of the data

Finding interrelation among the large number of variables in terms of a few integrated or latent factors. Total variation is decomposed in variation due to common factors, due to individual specific factors and by errors

$$v\,(X_1, X_2, ...X_2) = af + U + e \tag{205}$$

Correlation matrix
R-factor analysis/Q-factor analysis
Sample size 10:1 to factors
Assumptions:

Underlying structure exists; Multicollinearity among variables
Bartlett test of sphericity; Measure of sampling adequacy
Factor extraction : latent root criterion, scree test criteria, percentage of variation criteria
Factor matrix : correlation; factor loading
Factor rotation: orthogonal - quatrimax or varimax
oblique : equimax
Reliability: Cronbach's alpha; cross loading

### 5.2.7 Cluster analysis

Grouping similar things together and finding the latent structure from more complex structure of multiple variables
Initial cluster solution
Minimum Euclidean distance and hierarchical procedure for cluster formation
Measurement of distance:
Euclidean distance
Squared Euclidian distance
City block
Chebychev distance
Mahalanobis distance
Proximity matrix and similarity index
Graphical approach : Dandograms, single linkage, complete linkage, centroid method
Non-heirarchical method: cluster seeds, sequential threshold, parallel threshold, optimisation
Combination of both hierarchical and non-heirarchical methods.
Stopping rule - agglomeration coefficient.

**Euclidian distance** Take an Euclidian space $\mathbb{R}^n$ with elements $x = (x_1, x_2, x_3, ....x_n)$

$$d(x,y) = \|x-y\| = \sqrt{(x_1-y_1)^2 + .... + (x_n-y_n)^2} \tag{206}$$

$d(x,y) \geq \sqrt{(x_j-y_j)^2} = \mid x_j - y_j \mid$ and $d(x,y) \leq \sum_{j=1}^{n} \mid x_j - y_j \mid$

### 5.2.8 Correspondence analysis (CA)

Expected vs expected values
Difference between and within rows and columns
Similarity analysis - measure of similarity
Optimal scaling or scoring
Reciprocal averaging
Homegeneity analysis

### 5.2.9 Multidimensional scaling

Identifying the dimension underlying the respondents evaluations
Price, quality;
Perceptual map and similarity scale; Stress measures

# 6 Econometric Analysis

### 6.0.10 Linear regression model

Regression analysis is a quantitative method used to find the line of best fit in the given data. X determines value of Y. It involves estimating the intercept and slope parameters that best fit the data.

- Consider a linear regression model:

$$Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i \qquad i = 1...N \tag{207}$$

Errors represent all missing elements from this relationship; plus and minuses cancel out.

$$\varepsilon_i \sim N\left(0, \sigma^2\right) \tag{208}$$

Null hypothesis: value of intercept and slope coefficients are zero.

$$H_0 : \beta_1 = 0$$
$$H_0 : \beta_2 = 0$$

Alternative hypotheses: Intercept and slope coefficients are non -zero.

$$H_A : \beta_1 \neq 0$$
$$H_A : \beta_2 \neq 0$$

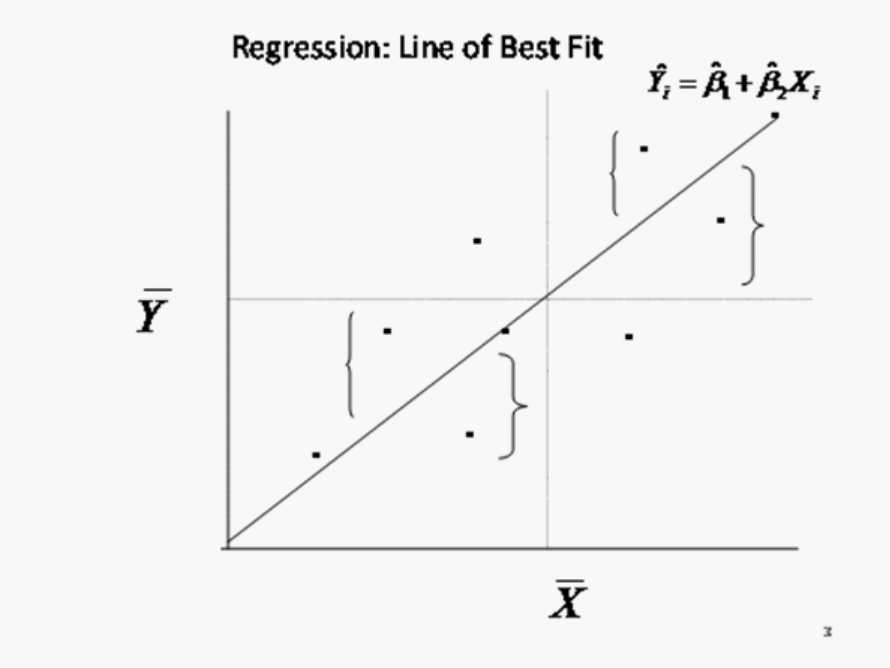
Parameter $\beta_2$ is slope, $\frac{\partial Y}{\partial X}$; it measures how much Y will change when X changes by one unit. Parameter $\beta_1$ is intercept. It shows amount of Y when X is zero.

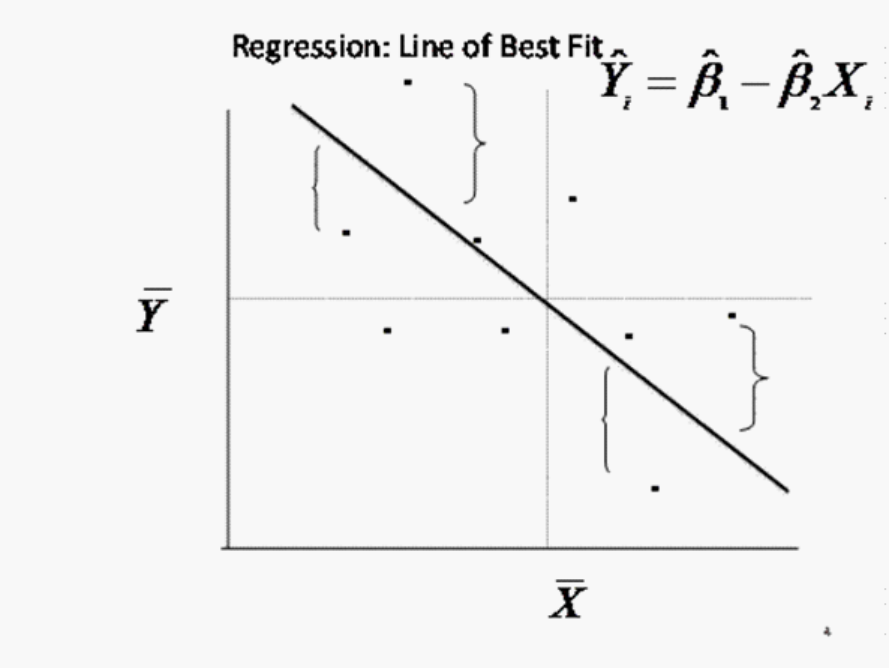
How to estimate parameters?

- Normal equations of above regression

$$\sum Y_i = \widehat{\beta}_1 N + \widehat{\beta}_2 \sum X_i \tag{209}$$

$$\sum Y_i X_i = \widehat{\beta}_1 \sum X_i + \widehat{\beta}_2 \sum X_i^2 \tag{210}$$




#### 6.0.11 OLS Estimators

Derivation of normal equations for the OLS estimators

Choose $\widehat{\beta}_1$ and $\widehat{\beta}_2$ to minimise sum of square errors:

$$\underset{\widehat{\beta}_1\widehat{\beta}_2}{Min}\ S = \sum \varepsilon_i^2 = \sum \left(Y_i - \widehat{\beta}_1 - \widehat{\beta}_2 X_{1,i}\right)^2 \tag{211}$$

First order conditions

$$\frac{\partial S}{\partial \widehat{\beta}_1} = 0; \frac{\partial S}{\partial \widehat{\beta}_2} = 0; \tag{212}$$

$$\sum \left(Y_i - \widehat{\beta}_1 - \widehat{\beta}_2 X_i\right)(-1) = 0 \tag{213}$$

$$\sum \left(Y_i - \widehat{\beta}_1 - \widehat{\beta}_2 X_i\right)(-X_i) = 0 \tag{214}$$

$$\sum Y_i = \widehat{\beta}_1 N + \widehat{\beta}_2 \sum X_i \tag{215}$$

$$\sum Y_i X_i = \widehat{\beta}_1 \sum X_i + \widehat{\beta}_2 \sum X_i^2 \tag{216}$$

There are two unknowns $\widehat{\beta}_1\widehat{\beta}_2$ and two equations. One way to find $\widehat{\beta}_1\widehat{\beta}_2$ is to use substitution and reduced form method.

Slope estimator by the reduced form equation method

Multiply the second equation by $N$ and first by $\sum X_i$

$$\sum X_i \sum Y_i = \widehat{\beta}_1 N \sum X_i + \widehat{\beta}_2 \left(\sum X_i\right)^2 \tag{217}$$

$$N \sum Y_i X_i = \widehat{\beta}_1 N \sum X_i + \widehat{\beta}_2 N \sum X_i^2 \tag{218}$$

By subtraction this reduces to

$$\sum X_i \sum Y_i - N \sum Y_i X_i = \widehat{\beta}_2 \left(\sum X_i\right)^2 - \widehat{\beta}_2 N \sum X_i^2 \tag{219}$$

$$\widehat{\beta}_2 = \frac{\sum X_i \sum Y_i - N \sum Y_i X_i}{\left(\sum X_i\right)^2 - N \sum X_i^2} = \frac{\sum x_i y_i}{\sum x_i^2} \tag{220}$$

This is the OLS Estimator of $\widehat{\beta}_2$, the slope parameter.

Intercept estimator by the reduced form equation method

When $\widehat{\beta}_2$ is known it is easy to find $\widehat{\beta}_1$ by averaging out the regression $Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i$ as:

$$\widehat{\beta}_1 = \overline{Y} - \widehat{\beta}_2 \overline{X} \tag{221}$$

Table 16: Y on X

| | Coefficient | Standard Error | t-value |
|---|---|---|---|
| Intercept | | | |
| X-Variable | | | |
| $R^2 =$ , $F =$ , $N =$ . | | | |

### 6.0.12 Homework:

Find $\widehat{\beta}_2$ and $\widehat{\beta}_1$ from the following data on y and X. Calculate predicted values $\widehat{Y}_i = \widehat{\beta}_1 + \widehat{\beta}_2 X_i$ and the predicted errors $\widehat{e}_i$.

Write results in a table

| DATA | | |
|---|---|---|
| Y | Contant | x |
| 4 | 1 | 5 |
| 6 | 1 | 8 |
| 7 | 1 | 10 |
| 8 | 1 | 12 |
| 11 | 1 | 14 |
| 15 | 1 | 17 |
| 18 | 1 | 20 |
| 22 | 1 | 25 |

$$\Longrightarrow \begin{bmatrix} Y \\ 4 \\ 6 \\ 7 \\ 8 \\ 11 \\ 15 \\ 18 \\ 22 \end{bmatrix} = \begin{bmatrix} X & \\ 1 & 5 \\ 1 & 8 \\ 1 & 10 \\ 1 & 12 \\ 1 & 14 \\ 1 & 17 \\ 1 & 20 \\ 1 & 25 \end{bmatrix} \begin{pmatrix} \widehat{\beta}_1 \\ \widehat{\beta}_2 \end{pmatrix} + \begin{bmatrix} e \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ e_7 \\ e_8 \end{bmatrix}$$

$$\begin{bmatrix} \sum Y_i = 91 \\ \sum Y_i X_i = 1553 \end{bmatrix} = \begin{bmatrix} N = 8 & \sum X_i = 111 \\ \sum X_i = 111 & \sum X_i^2 = 1843 \end{bmatrix}$$

$$\begin{bmatrix} \widehat{\beta}_1 \\ \widehat{\beta}_2 \end{bmatrix} = \begin{bmatrix} N & \sum X_i \\ \sum X_i & \sum X_i^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum Y_i \\ \sum Y_i X_i \end{bmatrix} \tag{222}$$

What is the variance of errors?

$$var(\ \widehat{e}_i) = E\left(\widehat{\varepsilon}_i^2\right) = \frac{\sum \widehat{e}_i^2}{N-k} = \widehat{\sigma}^2 \tag{223}$$

What is $R^2 = \frac{\sum \widehat{y}_i^2}{\sum y_i^2}$ and $F = \frac{RSS/(K-1)}{ESS/(N-k)}$ statistics and $\overline{R}^2 = 1 - (1-R^2)\frac{N-1}{N-K}$.

What is the variance of estimated parameters?

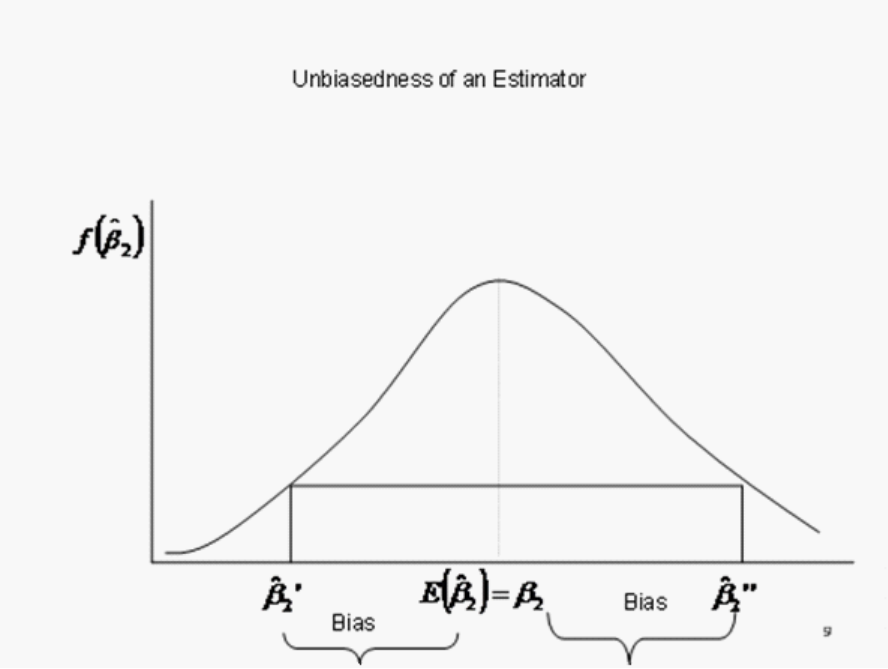
$$Var\left(\widehat{\beta}_2\right) = var\left[\frac{\sum\left(X_i - \overline{X}\right)}{\sum\left(X_i - \overline{X}\right)^2}\right] var\left(y_i\right) = \frac{1}{\sum x_i^2}\widehat{\sigma}^2 \tag{224}$$

$$var\left(\widehat{\beta}_1\right) = \left[\frac{1}{N} + \frac{\overline{X}^2}{\sum x_i^2}\right]\widehat{\sigma}^2 \tag{225}$$

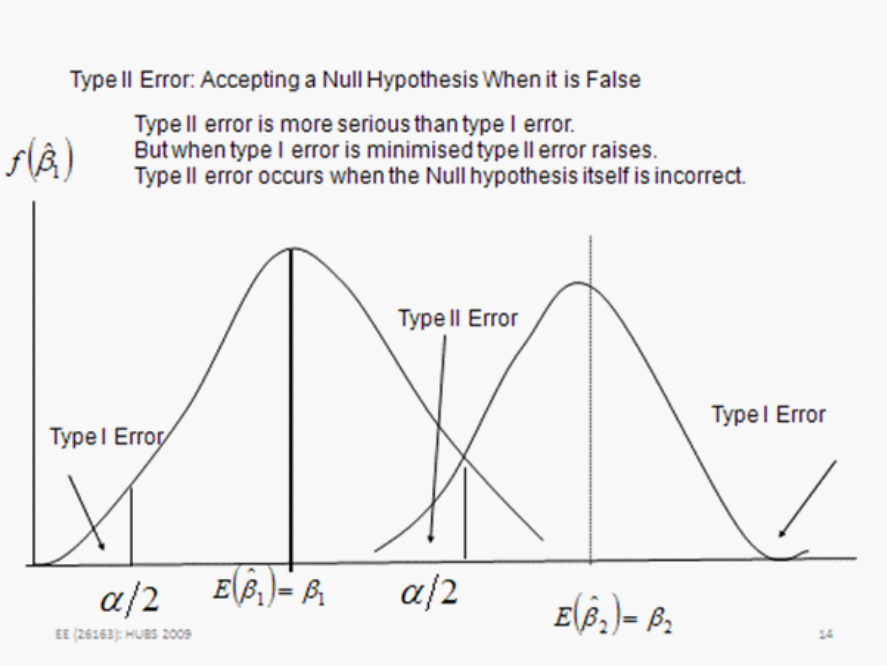
$$SE\left(\widehat{\beta}_2\right) = \sqrt{Var\left(\widehat{\beta}_2\right)}\ and SE\left(\widehat{\beta}_1\right) = \sqrt{Var\left(\widehat{\beta}_1\right)} \tag{226}$$

and correpsonding t scores

$$t_{\widehat{\beta}_2} = \frac{\widehat{\beta}_2 - \beta_2}{SE\left(\widehat{\beta}_2\right)} \; and \; t_{\widehat{\beta}_1} = \frac{\widehat{\beta}_1 - \beta_1}{SE\left(\widehat{\beta}_1\right)} \tag{227}$$




Type I and Type II errors in regression estimations




## 6.1 Multiple Regression

Consider a multiple linear regression model:

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \beta_3 X_{3,i} + .... + \beta_k X_{k,i} + \varepsilon_i \quad i = 1 \ ...N \tag{228}$$

Assumptions:

$$E(\varepsilon_i) \ = 0 \tag{229}$$

$$E(\varepsilon_i x_{j,i}) \ = 0; \ var(\varepsilon_i) \ = \sigma^2 \quad for \ \forall \ i; \ \varepsilon_i \tilde{} N(0, \sigma^2) \tag{230}$$

$$covar(\varepsilon_i \varepsilon_j) \ = 0 \tag{231}$$

Explanatory variables are not correlated.

$$E(X_{1,i} X_{1,j}) = 0 \tag{232}$$

Objective is to choose parameters that minimise the sum of squared errors

$$\underset{\widehat{\beta}_0 \widehat{\beta}_1 \widehat{\beta}_2 ... \widehat{\beta}_k}{Min} S = \sum \varepsilon_i^2 = \left( Y_i - \widehat{\beta}_0 - \widehat{\beta}_1 X_{1,i} - \widehat{\beta}_2 X_{2,i} - \widehat{\beta}_3 X_{3,i} - .... - \widehat{\beta}_k X_{k,i} \right)^2 \tag{233}$$

Derivation of Normal Equations

$$\frac{\partial S}{\partial \widehat{\beta}_0} = 0; \frac{\partial S}{\partial \widehat{\beta}_1} = 0; \frac{\partial S}{\partial \widehat{\beta}_2} = 0; \frac{\partial S}{\partial \widehat{\beta}_3} = 0; ..... \frac{\partial S}{\partial \widehat{\beta}_k} = 0 \tag{234}$$

Normal equations for two explanatory variable case

$$Y_i = \widehat{\beta}_0 + \widehat{\beta}_1 X_{1,i} + \widehat{\beta}_2 X_{2,i} + \varepsilon_i \tag{235}$$

$$\sum Y_i = \widehat{\beta}_0 N + \widehat{\beta}_1 \sum X_{1,i} + \widehat{\beta}_2 \sum X_{2,i} \tag{236}$$

$$\sum X_{1,i} Y_i = \widehat{\beta}_0 \sum X_{1,i} + \widehat{\beta}_1 \sum X_{1,i}^2 + \widehat{\beta}_2 \sum X_{1,i} X_{2,i} \tag{237}$$

$$\sum X_{2,i} Y_i = \widehat{\beta}_0 \sum X_{2,i} + \widehat{\beta}_1 \sum X_{1,i} X_{2,i} + \widehat{\beta}_2 \sum X_{2,i}^2 \tag{238}$$

$$\begin{bmatrix} \sum Y_i \\ \sum X_{1,i} Y_i \\ \sum X_{2,i} Y_i \end{bmatrix} = \begin{bmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i} X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i} X_{2,i} & \sum X_{2,i}^2 \end{bmatrix} \begin{bmatrix} \widehat{\beta}_0 \\ \widehat{\beta}_1 \\ \widehat{\beta}_2 \end{bmatrix} \tag{239}$$

Normal equations in matrix form

$$\begin{bmatrix} \widehat{\beta}_0 \\ \widehat{\beta}_1 \\ \widehat{\beta}_2 \end{bmatrix} = \begin{bmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i} X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i} X_{2,i} & \sum X_{2,i}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum Y_i \\ \sum Y_i X_{1,i} \\ \sum Y_i X_{2,i} \end{bmatrix} \tag{240}$$

$$\beta = (X'X)^{-1} X'Y \tag{241}$$

$$\widehat{\beta}_0 = \frac{\begin{vmatrix} \sum Y_i & \sum X_{1,i} & \sum X_{2,i} \\ \sum Y_i X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i} X_{2,i} \\ \sum Y_i X_{2,i} & \sum X_{1,i} X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}}{\begin{vmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i} X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i} X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}} \tag{242}$$

Cramer Rule to solve for parameters

$$\widehat{\beta}_1 = \frac{\begin{vmatrix} N & \sum Y_i & \sum X_{2,i} \\ \sum X_{1,i} & \sum Y_i X_{1,i} & \sum X_{1,i} X_{2,i} \\ \sum X_{2,i} & \sum Y_i X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}}{\begin{vmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i} X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i} X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}} \tag{243}$$

$$\widehat{\beta}_2 = \frac{\begin{vmatrix} N & \sum X_{1,i} & \sum Y_i \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum Y_i X_{1,i} \\ \sum X_{2,i} & \sum X_{1,i} X_{2,i} & \sum Y_i X_{2,i} \end{vmatrix}}{\begin{vmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i} X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i} X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}} \tag{244}$$

### 6.1.1 Homework: hypothesis testing and reporting results

Does level of unemployment depend on claimant count, strikes and work hours?

How does the level of unemployment ($Y_i$)relate to the level of claimant counts ($X_{1,i}$), numbers of stoppages($X_{2,i}$) because of industrial strikes and number of work hours ($X_{3,i}$)in UK? Use data in strike.xls from the Labour Force Survey for 19 years;$N = 19$.to compute a regression model of the form. Put the results in the given tables.

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \beta_3 X_{3,i} + \varepsilon_i \quad i = 1 \ldots N \tag{245}$$

Analysis of Variance

Table 17: Testing overall significance by F-test

| Source of Variance | Sum | Degrees of freedom | Mean | F-value |
|---|---|---|---|---|
| Total sum square (TSS) | | | | |
| Regression Sum Square (RSS) | | | | |
| Sum of square error | | | | |

Table of results summarising all above calculations are presented as:

Table 18: Determinants of Unemployment Rate

| | Coefficient | Standard Error | t-value |
|---|---|---|---|
| Intercept | | | |
| Claimant Count | | | |
| Stoppages | | | |
| Work hours | | | |
| $R^2 =$ , $F =$ , $N =$ | | | |

Dummy Variables in a Regression Model

- It represents qualitative aspect or characteristic in the data

  Quality : good, bad; Location: south/north/east/west; characteristics: fat/thin or tall/short

  Time: Annual 1970s/ 1990s.; seasonal: Summer,Autumn, Winter, Spring;

- Gender: male/female; Education: GCSE/UG/PD/PhD

  Subjects: Math/English/Science/Economics

- Ethnic backgrounds: Black, White, Asian, Cacasian, European, American, Latinos, Mangols, Ausis.

$$Y_i = \beta_1 + \beta_2 X_i + \beta_2 D_i + \varepsilon_i \quad i = 1 \ldots N \tag{246}$$

$$\varepsilon_i \sim N\left(0, \sigma^2\right) \tag{247}$$

- Here $D_i$ is special type of variable

$$D_i = \int \begin{array}{l} 1 = \text{if the certain quality exists} \\ 0 = \text{otherwise} \end{array} \tag{248}$$

### 6.1.2 Dummy Variables in a Regression Model

- Three types of dummy

  1. Slope dummy
  2. Intercept dummy
  3. Interaction between slope and intercept
     Examples
     - Earning differences by gender, region, ethnic origin or religion, occupation, education level.
     - Unemployment duration by gender, region, ethnic origin or religion, occupation, education level.
     - Demand for a product by weather, season, gender, region, ethnicity or religion, occupation, education level.
     - Test scores by gender, previous background, ethnic origin
     - Growth rates by decades, countries, exchange rate regimes

Dummy Variables Trap: Consider seasonal dummies as

$$Y_i = \beta_1 + \beta_2 X_i + \beta_2 D_1 + \beta_2 D_2 + \beta_2 D_3 + \beta_2 D_4 + \varepsilon_i \tag{249}$$

where

$$D_1 = \int \begin{array}{l} 1 = \text{if summer} \\ 0 \text{ otherwise} \end{array} \tag{250}$$

$$D_2 = \int \begin{array}{l} 1 = \text{if autumn} \\ 0 \text{ otherwise} \end{array} \tag{251}$$

$$D_3 = \int \begin{array}{l} 1 = \text{if winter} \\ 0 \text{ otherwise} \end{array} \tag{252}$$

$$D_4 = \int \begin{array}{l} 1 = \text{if spring} \\ 0 \text{ otherwise} \end{array} \tag{253}$$

- Since $\sum D_i = 1$, it will cause multicollinearity as:

$$D_1 + D_2 + D_3 + D_4 = 1 \tag{254}$$

  drop on of $D_i$ to avoid the dummy variable trap.

Dummy Variables in a piecewise linear regression models

- Threshold effects in sales
- tariff charges by volume of transaction -mobile phones
- Panel regression: time and individual dummies
- Pay according to hierarchy in an organisation

- profit from whole sale and retail sales
- age dependent earnings -Scholarship for students, pensions and allowances for elderly
- tax allowances by level of income or business
- Investment credit by size of investment
- prices, employments, profits or sales for small, medium and large scale corporations
- requirements according to weight or height of body

Chow Test for stability of parameters or structural change

- Use n1 and n2 observations to estimate overall and separate regressions with (n1+n2-k, n1-k, and n2-k) degrees of freedoms;
- obtain SSR1(with n1+n2-k dfs),
- SSR2 (with n1-k dfs),
- SSR3 (with n2-k dfs) and
- SSR4 = SSR1+ SSR2 (with n1+n2-2k dfs),
- obtain S5 = S1-S4;
- do F-test

$$F = \frac{\frac{S_5}{k}}{\frac{S_5}{(n1+n2-2k)}} \tag{255}$$

The advantage of this approach to the Chow test is that it does not require the construction of the dummy and interaction variables.

## 6.2 Multicollinearity

### 6.2.1 Exact mulitcollinearity and breakdown of OLS

Exact collinearity happens when on variable is exactly correlated to another variable in a multiple regression model as:

Consider three variable regression

$$Y_i = \widehat{\beta}_0 + \widehat{\beta}_1 X_{1,i} + \widehat{\beta}_2 X_{2,i} + \varepsilon_i \tag{256}$$

$$\begin{bmatrix} \widehat{\beta}_0 \\ \widehat{\beta}_1 \\ \widehat{\beta}_2 \end{bmatrix} = \begin{bmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i}X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i}X_{2,i} & \sum X_{2,i}^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum Y_i \\ \sum Y_i X_{1,i} \\ \sum Y_i X_{2,i} \end{bmatrix} \tag{257}$$

$$X_{1,i} = \lambda X_{2,i} \tag{258}$$

In existence of exact multicollinearity $X'X$ is singular, i.e. $|X'X| = 0 \Longrightarrow (X'X)^{-1} = \infty$

Look at the formula's of parameters each is divided by $|X'X| = \begin{vmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i}X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i}X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}$

$$\widehat{\beta}_0 = \frac{\begin{vmatrix} \sum Y_i & \sum X_{1,i} & \sum X_{2,i} \\ \sum Y_i X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i}X_{2,i} \\ \sum Y_i X_{2,i} & \sum X_{1,i}X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}}{\begin{vmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i}X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i}X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}} \tag{259}$$

Use Cramer Rule to solve for parameters

$$\widehat{\beta}_1 = \frac{\begin{vmatrix} N & \sum Y_i & \sum X_{2,i} \\ \sum X_{1,i} & \sum Y_i X_{1,i} & \sum X_{1,i}X_{2,i} \\ \sum X_{2,i} & \sum Y_i X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}}{\begin{vmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i}X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i}X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}} \tag{260}$$

$$\widehat{\beta}_2 = \frac{\begin{vmatrix} N & \sum X_{1,i} & \sum Y_i \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum Y_i X_{1,i} \\ \sum X_{2,i} & \sum X_{1,i}X_{2,i} & \sum Y_i X_{2,i} \end{vmatrix}}{\begin{vmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i}X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i}X_{2,i} & \sum X_{2,i}^2 \end{vmatrix}} \tag{261}$$

Evaluate the determinant

$$|X'X| = \begin{vmatrix} N & \sum X_{1,i} & \sum X_{2,i} \\ \sum X_{1,i} & \sum X_{1,i}^2 & \sum X_{1,i}X_{2,i} \\ \sum X_{2,i} & \sum X_{1,i}X_{2,i} & \sum X_{2,i}^2 \end{vmatrix} \tag{262}$$

When $|X'X| = 0$ $\beta_0$,$\beta_1$,and $\beta_2$ cannot be calculated.

### 6.2.2 Variance inflation factor

Inexact multicollinearity occurs when there is high degree of collinearity among explanatory variables.

Let correlations between $X_{1,i}$ and $X_{2,i}$ be given by $r_{12}$. Then Variance inflation factor is $\frac{1}{(1-r_{12}^2)}$

$$
\begin{aligned}
var\left(\widehat{\beta}_2\right) &= \frac{\sum x_{1,i}^2}{\left[\sum x_{1,i}^2 \sum x_{2,i}^2 - \left(\sum x_{1,i}x_{2,i}\right)^2\right]}\sigma^2 \\
&= \frac{1}{\left[\frac{\sum x_{1,i}^2 \sum x_{2,i}^2}{\sum x_{1,i}^2} - \frac{\left(\sum x_{1,i}x_{2,i}\right)^2}{\sum x_{1,i}^2}\right]}\sigma^2 \\
&= \frac{1}{\sum x_{2,i}^2 \left[\frac{\sum x_{1,i}^2}{\sum x_{1,i}^2} - \frac{\left(\sum x_{1,i}x_{2,i}\right)^2}{\sum x_{2,i}^2 \sum x_{1,i}^2}\right]}\sigma^2 \\
&= \frac{1}{\sum x_{2,i}^2 \left[1 - r_{12}^2\right]}\sigma^2 \\
&= \frac{1}{(1-r_{12}^2)}\frac{1}{\sum x_{2,i}^2}\sigma^2
\end{aligned} \tag{263}
$$

Variance Inflation Factor in Inexact Multicollinearity

Let correlations between $X_{1,i}$ and $X_{2,i}$ be given by $r_{12}$. Then Variance inflation factor is $\frac{1}{(1-r_{12}^2)}$

$$
\begin{aligned}
var\left(\widehat{\beta}_1\right) &= \frac{\sum x_{2,i}^2}{\sum x_{1,i}^2 \sum x_{2,i}^2 - \left(\sum x_{1,i}x_{2,i}\right)^2}\sigma^2 \\
&\quad \frac{1}{\left[\frac{\sum x_{1,i}^2 \sum x_{2,i}^2}{\sum x_{2,i}^2} - \frac{\left(\sum x_{1,i}x_{2,i}\right)^2}{\sum x_{2,i}^2}\right]}\sigma^2 \\
&= \frac{1}{\sum x_{1,i}^2 \left[\frac{\sum x_{2,i}^2}{\sum x_{2,i}^2} - \frac{\left(\sum x_{1,i}x_{2,i}\right)^2}{\sum x_{1,i}^2 \sum x_{2,i}^2}\right]}\sigma^2 \\
&= \frac{1}{\sum x_{1,i}^2 \left[1 - r_{12}^2\right]}\sigma^2
\end{aligned} \tag{264}
$$

Solutions for Multicollinearity Problem

When Variance is high the standard errors are high and that makes t-statistics very small and insignificant

$$
\begin{aligned}
SE\left(\widehat{\beta}_2\right) &= \sqrt{var\left(\widehat{\beta}_2\right)}\ ;\ SE\left(\widehat{\beta}_1\right) = \sqrt{var\left(\widehat{\beta}_1\right)}\ ; \\
t_{\widehat{\beta}_1} &= \frac{\widehat{\beta}_1 - \beta_1}{SE\left(\widehat{\beta}_1\right)};\ t_{\widehat{\beta}_2} = \frac{\widehat{\beta}_2 - \beta_2}{SE\left(\widehat{\beta}_2\right)}
\end{aligned} \tag{265}
$$

.since $0 < r_{12} < 1$ it raises the variance and hence standard errors and lowers t-values.

**Remedial measures for multicollinearity**

1. First detect the pairwise correlations between explanatory variables such $X_{1,i}$ and $X_{3,i}$ be given by $r_{12}$.
2. Drop highly correlated variables.
3. Adopts Klein's rule of thumb:
4. Compare $R_y^2$ from overall regression to $R_x^2$ from auxiliary regression. Determine multicollinearity if $R_x^2 > R_y^2$. Drop highly correlated variables.

### 6.2.3 Homework: Multicollinearity

Fit a three variable regression $Y_i = \widehat{\beta}_0 + \widehat{\beta}_1 X_{1,i} + \widehat{\beta}_2 X_{2,i} + \varepsilon_i$ for the following data.

Table 19: Data for a multiple regression

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| y | 3 | 5 | 7 | 6 | 9 | 6 | 7 |
| x1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| x2 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |

Suppose you have the following data set on number of tickets sold in a football match ($Y$), price of tickets ($X_1$ ) and income of the customers ($X_2$). and Y are measured in 10 thousand pounds. You want to find out the exact relation between tickets sold and prices and income of people watching football games.

Table 20: Monthy charges and number of customers

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $X_{1,i}$ | 11 | 7 | 6 | 5 | 3 | 2 | 1 |
| $X_{2,i}$ | 2 | 2 | 4 | 5 | 6 | 5 | 4 |
| $Y_i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

1. (a) Write a simple regression model to explain the number of tickets sold in terms of the price of the ticket. Explain briefly underlying assumptions and expected signs of the parameters in this model.
   (b) Estimate the slope and intercept parameters. Use cross products and squared terms provided for you in the above table.
   (c) Using your estimates in (b) find the explained squared sum $\sum \widehat{y}_i^2$ , sum of squared errors $\sum \widehat{e}_i^2$ and the $R^2$and $\overline{R}^2$.
   (d) Estimate the variance of the error term and the slope coefficient. Explain its importance.
   (e) Test whether the slope term is significant at 5% confidence level.
   (f) Build 95 percent confidence interval for estimate of slope and intercept terms.
   (g) Discuss how reducing type I error may cause increase in type II errors.

(h) Calculate the elasticity of demand for football around the mean of $Y$ and $X_1$.

(i) Write a multiple regression model to explain the number of tickets sold in terms of the price of the ticket and the income of individuals going to the football game. What additional assumption(s) do you need while introducing an additional variable.

(j) Estimate the parameters of that multiple regression model.

(k) What is your prediction of the number of tickets sold if $X_1$= 5 and $X_2$=4?

(l) Introduce dummy variables in your multiple regression model to show differences in demand for football ticket based on gender differences (1 for male and 0 for females), four seasons (autumn, winter, spring and summer) and interaction between gender and income.

## 6.3 Heteroskedasticity

### 6.3.1 Causes

Heteroskedasticity occurs when variances of errors are not constant, $var(\varepsilon_i) \neq \sigma_i^2$ variance of errors vary for each $i$. This is mainly a cross section problem. Main reason for this are

- Learning reduces errors;
  - driving practice, driving errors and accidents
  - typing practice and typing errors,
  - defects in productions and improved machines
- Improved data collection: better formulas and goods software
- More heteroskedasticity exists in cross section than in time series data.

### 6.3.2 Consequences

OLS estimator is still unbiased but inefficient

$$Var\left(\widehat{\beta}_2\right) = var\left[\frac{\sum\left(X_i - \overline{X}\right)}{\sum\left(X_i - \overline{X}\right)^2}\right] var\left(y_i\right) = \frac{\sum x_i^2 \sigma_i^2}{\left[\sum x_i^2\right]^2} \tag{266}$$

It should have been

$$Var\left(\widehat{\beta}_2\right) = \frac{1}{\left[\sum x_i^2\right]} \sigma^2 \tag{267}$$

larger value of variance makes coefficients insignificant and can lead to a wrong conclusion.

### 6.3.3 Detecting heteroskedasticity

Regress the model, find the estimates and plot error against X variables. See if it has any relations.
Various tests of heteroskedasticity

- Spearman Rank Test
- Park Test
- Goldfeld-Quandt Test
- Glesjer Test
- Breusch-Pagan,Godfrey test
- White Test
- ARCH test

**White test**
**White test of heteroskedasticity is more general test**
$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \varepsilon_i \quad i = 1 \ldots N$

- run OLS and obtain error squares $\widehat{e}_i^2$
- regress $\widehat{e}_i^2 = \alpha_0 + \alpha_1 X_{1,i} + \alpha_2 X_{2,i} + \alpha_3 X_{1,i}^2 + \alpha_4 X_{2,i}^2 + \alpha_5 X_{1,i} X_{2,i} + v_i$
- Compute test statistics $n.R^2 = \chi_{df}^2$
- If the calculated $\chi_{df}^2$ value is greater that the $\chi_{df}^2$ table value then, there is evidence of heteroskedasticity.

**Spearman rank test of heteroskedactity**

$$r_s = 1 - 6 \times \frac{\sum_i d_i^2}{n\left(n^2 - 1\right)} \tag{268}$$

- steps:
- run OLS of y on x.
- obtain errors e
- rank e and y or x
- find the difference of the rank
- use t-statistics if ranks are significantly different assuming n>8 and rank correlation coefficient $\rho$=0.

$$t = 1 - 6 \times \frac{r_s\sqrt{n-2}}{\sqrt{1 - r_s^2}} \quad with \quad df \quad (n-2) \tag{269}$$

- If $t_{cal} > t_{crit}$ there is heteroskedasticity.

GLS Solution of the Heteroskedasticity Problem When Variance is Known

$$\frac{Y_i}{\sigma_i} = \frac{\beta_1}{\sigma_i} + \beta_2 \frac{X_i}{\sigma_i} + \frac{\varepsilon_i}{\sigma_i} \quad i = 1 \ldots N \tag{270}$$

Variance with this transformation equals 1. $var\left(\frac{\varepsilon_i}{\sigma_i}\right) = \frac{\sigma_i^2}{\sigma_i^2} = 1$
if

$$\sigma_i^2 = \sigma^2 X_i \tag{271}$$

$$\frac{Y_i}{X_i} = \frac{\beta_1}{X_i} + \beta_2 + \frac{\varepsilon_i}{X_i} \; ; \quad var\left(\frac{\varepsilon_i}{x_i}\right) = \frac{\sigma^2 x_i^2}{x_i^2} = \sigma^2 \tag{272}$$

In matrix notation

$$\beta_{OLS} = \left(X'X\right)^{-1}\left(X'Y\right) \tag{273}$$

$$\beta_{GLS} = \left(X'\Omega^{-1}X\right)^{-1}\left(X'\Omega^{-1}Y\right) \tag{274}$$

$\Omega^{-1}$ is inverse of variance covariance matrix.

## 6.4 Autocorrelation

Consider a linear regression

$$Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t \quad t = 1 \ldots T \tag{275}$$

Classical assumptions

$$E\left(\varepsilon_t\right) \; = 0 \tag{276}$$

$$E\left(\varepsilon_t x_{_t}\right) \; = 0 \tag{277}$$

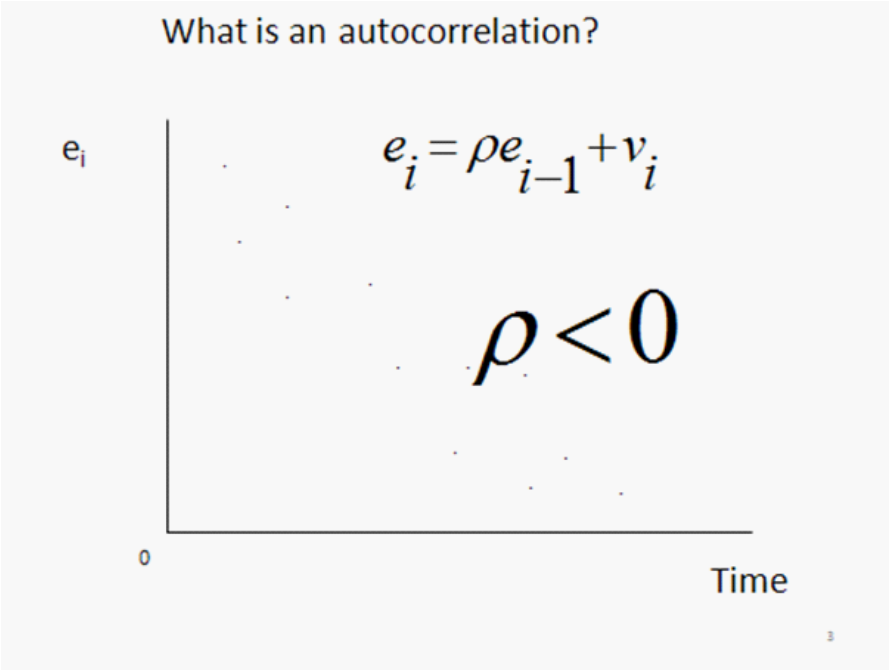
$$var\left(\varepsilon_t\right) = \sigma^2 \quad for \; \forall \; t \;\; covar\left(\varepsilon_t \varepsilon_{t-1}\right) = 0 \tag{278}$$

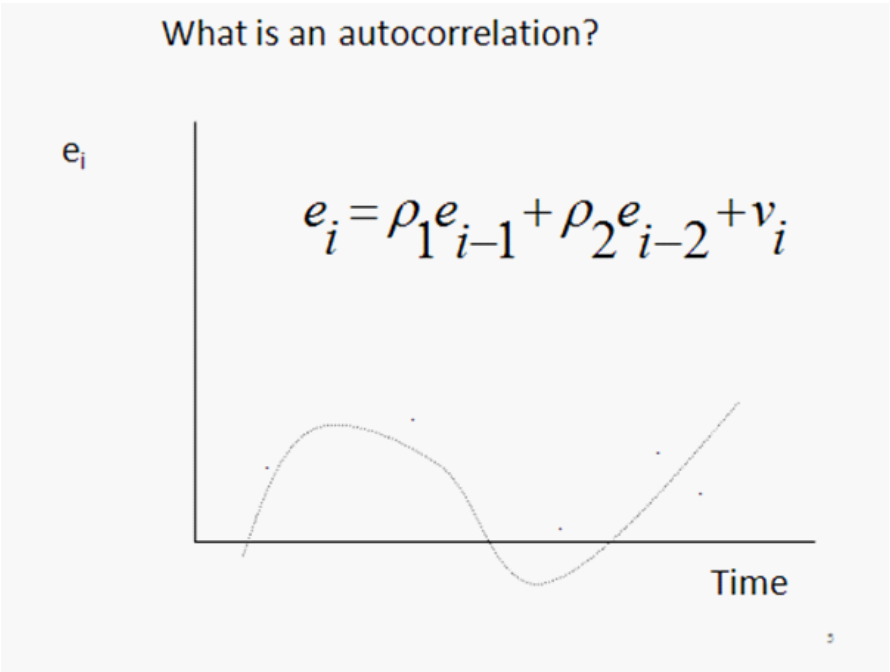
### 6.4.1 Nature and causes

In presence of autocorrelation (first order)

$$\varepsilon_t = \rho \varepsilon_{t-1} + v_t \tag{279}$$

Autocorrelation occurs when covariances of errors are not zero, $covar\left(\varepsilon_t \varepsilon_{t-1}\right) \neq 0$ covariance of errors are nonnegative This is mainly a problem observed in time series data.




**Causes of autocorrelation**

- inertia , specification bias, cobweb phenomena
- manipulation of data

**Consequences of autocorrelation**

1. (a) Estimators are still linear and unbiased, but
   (b) they there not the best, they are inefficient.

Remedial measures

1. (a) When $\rho$ is known - transform the model
   (b) When $\rho$ is unknown estimate it and transform the model

### 6.4.2 Consequences of autocorrelation

**Consequences of autocorrelation**

- Estimators are still linear and unbiased, but
- they there not the best, they are inefficient because variance of parameters with autocorrelation becomes

$$Var\left(\widehat{\beta}_2\right) = \frac{1}{\sum x_t^2}\sigma^2 + 2\frac{\sum w_t w_{t-1}}{\left[\sum x_t^2\right]^2}\rho^s \tag{280}$$

but not

$$Var\left(\widehat{\beta}_2\right) = \frac{1}{\left[\sum x_i^2\right]}\sigma^2 \tag{281}$$

### 6.4.3 Durbin-Watson test of autocorrelation

$$d = \frac{\sum\limits_{t=1}^{T}\left(e_t - e_{t-1}\right)^2}{\sum\limits_{t=1}^{T} e_t^2} \tag{282}$$

$$d = \frac{\sum\limits_{t=1}^{T}\left(e_t^2 - 2e_t e_{t-1} + e_{t-1}^2\right)}{\sum\limits_{t=1}^{T} e_t^2} = 2\left(1-\rho\right) \tag{283}$$

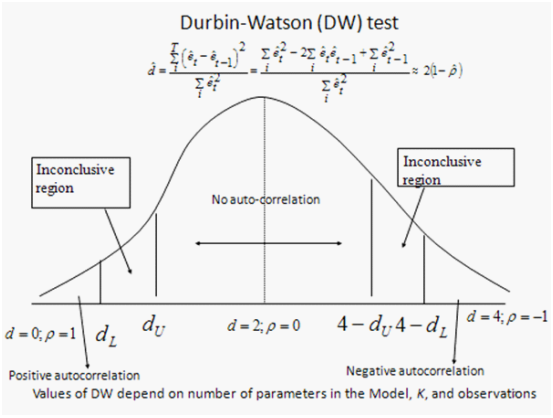
Autocorrelation and Durbin-Watson Statistics

$$d = 2\left(1-\rho\right) \tag{284}$$

$$\rho = 0 \Longrightarrow d = 2 \tag{285}$$

$$\rho = -1 \Longrightarrow d = 4 \tag{286}$$

**Durbin-Watson Distribution**




### 6.4.4 Remedial measures

Transformation of the model in the presence of autocorrelation

when autocorrelation coefficient is known

$$Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t \quad t = 1 \ldots T \tag{287}$$

$$\varepsilon_t = \rho \varepsilon_{t-1} + v_t \tag{288}$$

$$Y_t - \rho Y_{t-1} = (\beta_1 - \rho\beta_1) + \beta_2 (X_t - \rho X_{t-1}) + \varepsilon_t - \rho\varepsilon_{t-1} \tag{289}$$

$$Y_t^* = \beta_1^* + \beta_2 X_t^* + \varepsilon_t^* \tag{290}$$

Apply OLS in this transformed model $\beta_1^*$ and $\beta_2$ will have BLUE properties.

When autocorrelation coefficient is unknown, this method is similar to the above ones, except that it involves multiple iteration for estimating $\rho$ . Steps are as following:

1. Get estimates $\widehat{\beta}_1$ and $\widehat{\beta}_2$ from the original model; get error terms $\widehat{e}_i$ and estimate $\widehat{\rho}$
2. Transform the original model multiplying it by $\widehat{\rho}$ and by taking the first difference,
3. Estimate $\widehat{\widehat{\beta}}_1$ and $\widehat{\widehat{\beta}}_2$ from the transformed model and get errors $\widehat{\widehat{e}}_i$ of this transformed model
4. Then again estimate $\widehat{\widehat{\rho}}$ and use those values to transform the original model as

$$Y_t - \widehat{\rho} Y_{t-1} = (\beta_1 - \widehat{\rho}\beta_1) + \beta_2 (X_t - \widehat{\rho} X_{t-1}) + \varepsilon_t - \widehat{\rho}\varepsilon_{t-1} \tag{291}$$

5. Continue this iteration process until $\widehat{\widehat{\rho}}$ converges.

PcGive suggests using differences in variables. Diagnos /ACF options in OLS in Shazam will generate these iterations.

GLS to solve autocorrelation

In matrix notation

$$\beta_{OLS} = (X'X)^{-1}(X'Y) \tag{292}$$

$$\beta_{GLS} = \left(X'\Omega^{-1}X\right)^{-1}\left(X'\Omega^{-1}Y\right) \tag{293}$$

$\Omega^{-1}$ is inverse of variance covariance matrix.

## 6.5 Homework on autocorrelation

1. Estimate consumption and import function from the data in macro.csv. Detect autocorrelation and suggest a correction procedure.
2. Run a regression of quarterly series of GDP on consumption; determine looking DW-statitstics whether there is an autocorrelation.
3. Follow up the remedial procedure and establish true relationship if you observe any autocorrelation.

## 6.6 Time series analysis

A time series is a data generating process and it is characterised by its mean variance and autocovariance. Trends, cycles, seasonality and irregular components of a time series is often represented in terms of autoregressive or moving average models or combinations of these. What is the right order of autoregressive or moving average component and combination of these two and what sorts of autocorrelation coefficients are required for stationary series and how can non-stationary series be made stationary and used for economic analysis is one the major topic in univariate time series analyses.

Multivariate vector autoregressive (VAR) and simultaneous equation models are used to analysis economic systems. These involve a number of equations with various time series variables. What is the long run relationship among variables how do adjustments occur towards the long run whenever these process deviate from the long run equilibrium and how economic variables can be coitegrated are discussed in multivariate time series analyses.

Time series models aim to explain the data generating process for $\{y_t\}_{-\infty}^{\infty} = \{y_{-\infty}.....y_{-1}.y_0.y_1.y_2.....y_T.y_{T+1}.y_{T+1}...$

A Time series consists of trend, cycle, season and irregular component

$$Y = T \times C \times S \times I \tag{294}$$

In a simple method the moving average gives $T \times C$ components and is used to isolate the $S \times I$ components. For instance for a 12 monthly moving average

$$\overline{Y}_i = \frac{1}{12}(Y_1 + Y_2 + ..... + Y_{12}) \tag{295}$$

$$S \times I = \frac{T \times C \times S \times I}{T \times C} = \frac{Y_i}{\overline{Y}_i} = z_t \tag{296}$$

Now to isolate the Irregular component $I$ from $S \times I$ take out the seasonal elements from $z_t$ assuming monthly data for 5 years (60 observations) compute the seasonal indices as following:

$$Month1 : \overline{z}_1 = \frac{1}{5}\left(z_1 + z_{13} + z_{25} + z_{39} + z_{48}\right) \tag{297}$$

$$Month2 : \overline{z}_2 = \frac{1}{5}\left(z_2 + z_{14} + z_{26} + z_{40} + z_{49}\right) \tag{298}$$

$$Month3 : \overline{z}_3 = \frac{1}{5}\left(z_3 + z_{15} + z_{26} + z_{41} + z_{50}\right) \tag{299}$$

$$.....$$

$$Month11 : \overline{z}_{11} = \frac{1}{5}\left(z_{11} + z_{23} + z_{35} + z_{47} + z_{59}\right) \tag{300}$$

$$Month12 : \overline{z}_{12} = \frac{1}{5}\left(z_{12} + z_{24} + z_{36} + z_{46} + z_{60}\right) \tag{301}$$

Deseasonalisation of data $Y_i^d = \frac{Y_i}{\overline{z}_i}$ and irregular component should be $i = \frac{z_t}{\overline{z}_i}$.
Trends:
Simple extrapolation

$$Y_t = c_1 + c_2 t \tag{302}$$

Exponential growth

$$Y_t = Ae^{rt} \tag{303}$$

Autoregressive model

$$Y_t = c_1 + c_2 Y_{t-1} \tag{304}$$

Log trend

$$\ln\left(Y_t\right) = c_1 + c_2 \ln\left(Y_{t-1}\right) \tag{305}$$

Quadratic trends:

$$Y_t = c_1 + c_2 t + c_3 t^2 \tag{306}$$

Logistic trend:

$$Y_t = \frac{1}{k + bt} \quad b > 1 \tag{307}$$

$$Y_t = e^{k_1 - \frac{k_2}{t}} \tag{308}$$

$$ln\left(Y_t\right) = k_1 - \frac{k_2}{t} \tag{309}$$

auto lagged with declining weights $\alpha < 1$

$$Y_t = \alpha Y_{t-1} + \alpha\left(1-\alpha\right) Y_{t-2} + \alpha\left(1-\alpha\right)^2 Y_{t-2} + ... + \alpha\left(1-\alpha\right)^n Y_{t-2} \tag{310}$$

Forecasting forward with these models is obvious.

## 6.7 Time series process

Simplest of these is a trend model

$$Y_t = \beta t + \varepsilon_t \tag{311}$$

with mean $E(Y_t) = \beta t$ and variance $E\left(Y_t - \beta t\right)^2 = E\left(\varepsilon_t\right)^2 = \sigma_\varepsilon^2$

Or it could have been just a constant plus a Gaussian white noise $\varepsilon_t \tilde{} N\left(0, \sigma^2\right)$ as:

$$Y_t = \mu + \varepsilon_t \tag{312}$$

with mean $E(Y_t) = \mu$ and variance $E\left(Y_t - \mu\right)^2 = E\left(\varepsilon_t\right)^2 = \sigma_\varepsilon^2$

Autocovariance of $\{y_t\}_{-\infty}^{\infty}$ for I realisations is

$$\gamma_{tj} = E\left(Y_t - \mu\right) E\left(Y_{t-j} - \mu\right) = E\left(\varepsilon_t\right) E\left(\varepsilon_{t-j}\right) = 0 \quad for\ j \neq 0 \tag{313}$$

Stationarity

when neither mean $\mu$ nor the autocovariance $\gamma_{ij}$ depend on time t then the $Y_t$ is covariance stationary or weakly stationary.

$$E(Y_t) = \mu\ for\ \forall\ t \tag{314}$$

$$E\left(Y_t - \mu\right) E\left(Y_{t-j} - \mu\right) = \gamma_j \quad for \text{ any } t \ \ and\ j = \begin{cases} \sigma_\varepsilon^2 & for\ j=0 \\ 0 & for\ j \neq 0 \end{cases} \tag{315}$$

For instance 312 is stationary while 311 not covariance stationary because its mean $\beta t$ is function of time.

If the process is stationary $\gamma_j$ is the same for any value of $t$ $\gamma_j = \gamma_{-j}$

$$\gamma_j = E\left(Y_{t+j} - \mu\right) E\left(Y_{(t+j)-j} - \mu\right) = E\left(Y_{t+j} - \mu\right) E\left(Y_t - \mu\right) = E\left(Y_t - \mu\right) E\left(Y_{t+j} - \mu\right) \ = \gamma_{-j} \tag{316}$$

### 6.7.1 Stationarity

Application of OLS with non-stationary variables generates a spurious regression, with a high R2 and very low Durbin-Watson statistics (R2>D). Many economic variables such as GDP, GDP components (C, Y, I, G and X), inflation, exchange rates, labour force evolve over time. It is important to check whether these series have a constant mean and constant variance before they can be used in regression analysis. A meaningful cause and effect relationship requires that the concerned series are stationary. Application of OLS procedure in non stationary series produces a spurious relationship. A spurious relationship implies significant test-statistics (t, f, chi-square, R-square) even though there is no relationship among the variables. Econometric estimation using non-stationary variables may generate meaningless result though it may apparently seem statistically significant.

What is a stationary variable?

When its mean and variance are constant.

$$E\left(Y_t\right) = \mu \tag{317}$$

$$var\left(Y_t\right) = \sigma^2 \tag{318}$$

When mean and variances are not constant, that variable is non-starionary, for instance a random walk

$$Y_t = Y_{t-1} + \varepsilon_i \quad t = 1 \ldots T \tag{319}$$

In an autoregressive model

$$Y_t = \rho Y_{t-1} + \varepsilon_i \quad t = 1 \ldots T \tag{320}$$

if the autocorrelation coefficient $\rho = 1$ then it becomes a random walk. This variable is non-stationary.

$$Y_t = \sum_{s=1}^{\infty} \rho^s \varepsilon_{t-s} \tag{321}$$

Current realisations are accumulation of past errors.
Prove that variance of this is .

$$var\left(Y_t\right) = t.\sigma^2 \tag{322}$$

Regression among non-stationary variables becomes spurious unless they are cointegrated.

### 6.7.2 Unit root and order of integration

A Non-Stationary variable can be made stationary by taking first difference as:

$$\Delta Y_t = Y_t - Y_{t-1} \tag{323}$$

If a variable becomes stationary by taking the first difference it is said to be integrated of order one

$$I\left(1\right) \tag{324}$$

If it becomes stationary after differencing $d$ time then it is called $I\left(d\right)$ variable.
Dickey-Fuller and Phillip-Perron unit root tests are used to determine stationarity of a variable.

$$Y_t = \rho Y_{t-1} + \varepsilon_i \tag{325}$$

### 6.7.3 Level, drift, trend and lag terms in unit root test

Dickey-Fuller and Phillip-Perron unit root tests are used to determine stationarity of a variable.

$$Y_t = \rho Y_{t-1} + \varepsilon_i \tag{326}$$

$$\Delta Y_t = (\rho - 1) Y_{t-1} + \varepsilon_i; \qquad \Delta Y_t = \gamma Y_{t-1} + \varepsilon_i; \tag{327}$$

Random walk with drift

$$\Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \varepsilon_i \tag{328}$$

trend stationary

$$\Delta Y_t = \alpha_0 + \alpha_1 t + \gamma Y_{t-1} + \varepsilon_i \tag{329}$$

Augmented Dickey-Fuller test

$$\Delta Y_t = \alpha_0 + \alpha_1 t + \gamma Y_{t-1} + \sum_{i=1}^{m} \rho^s \Delta Y_{t-i} + \varepsilon_i \tag{330}$$

Cointegration in a regression

$$Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t \tag{331}$$

First do the regression and then estimate the error as

$$\widehat{\varepsilon}_t = Y_t - \widehat{\beta}_1 - \widehat{\beta}_2 X_t \tag{332}$$

$Y_t$ and $X_t$ are cointegrated if the estimated error is stationary $\widehat{\varepsilon}_t \sim I(0)$

$$\widehat{\varepsilon}_t = \rho \widehat{\varepsilon}_{t-1} + \varepsilon_t \tag{333}$$

if $\rho < 1$ the error $\widehat{\varepsilon}_t$ is stationary and $Y_t$ and $X_t$ are cointegrated. They have a long run relationship.

When variables are cointegrated there is an error correction mechanism.

$$Y_t = \varphi_2 X_t + \epsilon_t \tag{334}$$

$$Y_t = X_t + \epsilon_t \; ; \qquad \varphi_2 = 1 \tag{335}$$

**Cointegration: Engle-Granger Representation Theorem**

$$\epsilon_t \ = Y_t - X_t \tag{336}$$

For test of cointegration

$$\Delta\epsilon_t \ = \gamma\epsilon_{t-1} \ + u_t \tag{337}$$

$$\Delta\left(Y_t - X_t\right) = \gamma\left(Y_{t-1} - X_{t-1}\right) + u_t \tag{338}$$

$$\Delta Y_t = \Delta X_t + \gamma\left(Y_{t-1} - X_{t-1}\right) + u_t \tag{339}$$

This is an error correction model. Term $\gamma\left(Y_{t-1} - X_{t-1}\right)$ gives the adjustment towards the long run equilibrium and $\Delta X_t$ denotes the short run impact.

$H_0$ : No cointegration; t- statistics can be used instead of DF test in error correction model.

**Granger Causality Test** Estimate the following model where $M_t$ is money $Y_t$ is GDP and test the causality as below:

$$Y_t = \sum_{i=1}^{n}\alpha_i M_{t-i} + \sum_{j=1}^{m}\beta_j Y_{t-j} + u_{1,t} \tag{340}$$

$$M_t = \sum_{i=1}^{n}\lambda_i M_{t-i} + \sum_{j=1}^{m}\delta_j Y_{t-j} + u_{2,t} \tag{341}$$

Unidirectional causality from $M_t$ to $Y_t$ requires $\sum_{i=1}^{n}\alpha_i \neq 0$ and $\sum_{j=1}^{m}\delta_j = 0$

Unidirectional causality from $Y_t$ to $M_t$ requires $\sum_{j=1}^{m}\delta_j \neq 0$ and $\sum_{i=1}^{n}\alpha_i = 0$

Bilateral causality between $Y_t$ to $M_t$ requires $\sum_{i=1}^{n}\alpha_i \neq 0$ and $\sum_{j=1}^{m}\delta_j \neq 0$

Independence of $Y_t$ to $M_t$ from each other $\sum_{i=1}^{n}\alpha_i = 0$ and $\sum_{j=1}^{m}\delta_j = 0$

Homework

1. Prove that mean of $Y_t = \delta + \rho y_{t-1} + \epsilon_t$, an AR(1) data generating process also includes an intercept is given by $\frac{\delta}{1-\rho}$and its variance is $\frac{\sigma^2}{1-\rho}$.
2. Prove that a random walk series is nonstationary.
3. Compute the mean and variance of ARMA(1,1) process.

### 6.7.4 Exercise

Stationarity, Unit Root and Cointegration

1. Study the monthly data on unemployment rate and inflation since 1972:1 to 2004:8 as given in "unmnth.xls" file. Use GiveWin PcGive to

- Draw diagrams to represent the rates of unemployment among males and females and the RPI over this period.
- Ascertain whether unit root exists in the overall unemployment rate, URT and RPI at 5% and 1% level of significance in level, in log and in the first difference of these series.
- Detrend the data with Hodrik-Prescott filter and conduct stochastic volatility tests.

2. Regress unemployment rate on inflation rate in levels and in the first differences. Test whether these series are cointegrated using the Engle-Granger procedure. (hint: stationarity of residuals).

3. The time series and represent the underlying data generating processes (DGP) of consumption $\{C_t\}$ and income $\{Y_t\}$. Answer the following questions regarding the properties these series.

   (a) What is meant by saying that$\{C_t\}$ and $\{Y_t\}$ are stationary series? Why is it important that the series are stationary for a robust regression analysis?

   (b) How do you determine whether $\{C_t\}$ and $\{Y_t\}$ are stationary series, or not?

   (c) Analyse the properties of these series when they follow a random walk, or have a unit root.

   (d) What is the meaning of the order of integration in this respect? Discuss any three different methods of checking for stationarity.

   (e) What is the meaning of cointegration between the series and ? How would you decide whether these series$\{C_t\}$ and $\{Y_t\}$ are cointegrated, or not?

   (f) If the original series $\{C_t\}$ and $\{Y_t\}$ are not co-integrated, what transformation can be applied to achieve co-integration? How do you decide the order of co-integration?

   (g) Use time series of consumption and income contained in Quarterly_cons.xls. Determine the order of integration for both consumption and income. Is there an evidence of cointegration between consumption and income in levels or in the first differences?

## 6.8 Simultaneous equation system

Main Features of Simultaneous Equation System

- Single equation models have Y dependent variables to be determined by a set of X variables and the error term.
- one way causation from independent variables to the dependent variables.
- However, many variables in economics are interdependent and there is two way causation.
- Consider a market model with demand and supply.
- Price determines quantity and quantity determines price.
- Same is true in national income determination model. Consumption and income.

Main Feature of Simultaneous Equation System

- Both quantities and prices and income and consumption are determined simultaneously.

- A system of equations, not a single equation, need to be estimated in order to be able to capture this interdependency among variables.
- The main features of a simultaneous equation model are:

  (i) two or more dependent (endogenous) variables; a number of exogenous variables

  (ii) a set of equations
- Computationally cumbersome, highly non-linearity in parameters and errors in one equation transmitted through the whole system

Identification issue in a Market Model

- Consider a relation between quantity and price

$$Q_t = \alpha_0 + \alpha_1 P_t + u_t \tag{342}$$

- A priory it is impossible to say whether this is a demand or supply model, both of them have same variables.
- If we estimate a regression model like this how can we be sure whether the parameters belong to a demand or supply model?
- We need extra information. Economic theory suggests that demand is related with income of individual and supply may respond to cost or weather condition; e.g. lagged price level $P_{t-1}$.

### 6.8.1 Market Model

$$Q_t^d = \alpha_0 + \alpha_1 P_t + \alpha_2 I_t + u_{1,t} \tag{343}$$

$$Q_t^s = \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + u_{2,t} \tag{344}$$

Market Model: Reduced form

$$Q_t = \Pi_{1,0} + \Pi_{1,1} P_{t-1} + \Pi_{1,2} I_t + V_{1,t} \tag{345}$$

$$P_t = \Pi_{2,0} + \Pi_{2,1} P_{t-1} + \Pi_{2,2} I_t + V_{2,t} \tag{346}$$

$$\begin{aligned} \Pi_{1,0} &= \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} \quad \Pi_{1,1} = \frac{-\alpha_2}{\alpha_1 - \beta_1} \; \Pi_{1,2} = \frac{-\beta_2}{\alpha_1 - \beta_1} \; ; V_{1,t} = \frac{u_{2,t} - u_{1,t}}{\alpha_1 - \beta_1} \\ \Pi_{2,0} &= \frac{\alpha_1\beta_0 - \alpha_0\beta_1}{\alpha_1 - \beta_1} \; \Pi_{2,1} = \frac{\alpha_1\beta_0 - \alpha_0\beta_1}{\alpha_1 - \beta_1} \qquad \Pi_{2,2} = \frac{\alpha_2\beta_2}{\alpha_1 - \beta_1}; \end{aligned} \tag{347}$$

$$V_{1,t} = \frac{u_{2,t} - u_{1,t}}{\alpha_1 - \beta_1}; V_{2,t} = \frac{\alpha_1 u_{2,t} - \beta_1 u_{1,t}}{\alpha_1 - \beta_1} \tag{348}$$

### 6.8.2 Keynesian Model

$$C_t = \beta_0 + \beta_1 Y_t + u_t \tag{349}$$

$$Y_t = C_t + I_t \tag{350}$$

$\beta_0$ and $\beta_1$are structural parameters ; $Y_t$ and $C_t$ are endogenous variables and $I_t$ is exogenous variable.

In the income determination model (example 2) the reduced form is obtained by expressing C and Y endogenous variables in terms of I which is the only exogenous variable in the model.

$$C_t = \frac{\beta_0}{1-\beta_1} + \frac{\beta_1}{1-\beta_1} I_t + \frac{1}{1-\beta_1} u_{1,t} \tag{351}$$

$$Y_t = \frac{\beta_0}{1-\beta_1} + \frac{1}{1-\beta_1} I_t + \frac{1}{1-\beta_1} u_{1,t} \tag{352}$$

$$C_t = \Pi_{1,1} + \Pi_{1,2} I_t + V_{1,t} \tag{353}$$

$$Y_t = \Pi_{2,1} + \Pi_{2,2} I_t + V_{2,t} \tag{354}$$

Retrieving the structural parameters of the model:

$$\Pi_{1,1} = \frac{\beta_0}{\beta_1} \quad \Pi_{1,2} = \frac{\beta_1}{1-\beta_1} \qquad \Pi_{2,1} = \frac{\beta_0}{1-\beta_1} \qquad \Pi_{2,2} = \frac{1}{1-\beta_1} \tag{355}$$

Keynesian Model: Simultaneity Bias

$$\widehat{\beta}_1 = \frac{\sum c_t y_t}{\sum y_t^2} = \frac{\sum \left(C_t - \overline{C}\right) y_t}{\sum y_t^2} = \frac{\sum C_t y_t}{\sum y_t^2} \tag{356}$$

$$\widehat{\beta}_1 = \frac{\sum C_t y_t}{\sum y_t^2} = \frac{\sum \left(\beta_0 + \beta_1 Y_t + u_t\right) y_t}{\sum y_t^2} \tag{357}$$

$$cov(Y, e) = E\left(Y_t - E\left(Y_t\right)\right) E\left(u_t - E\left(u_t\right)\right) = E\left(\frac{u_t}{1-\beta_1}\right) u_t = \frac{\sigma_e^2}{1-\beta_1} \tag{358}$$

$$p\lim\left(\widehat{\beta}_1\right) = \beta_1 + \frac{\sum u_t \ y_t}{\sum y_t^2} = \beta_1 + \frac{\frac{\sum u_t \ y_t}{T}}{\frac{\sum y_t^2}{T}} = \beta_1 + \frac{\frac{\sigma_e^2}{1-\beta_1}}{\sigma_y^2} \tag{359}$$

Techniques of estimation of simultaneous equation models

- 1. Single Equations Methods: Recursive OLS
  2. Ordinary Least Squares
  3. Indirect Least Squares
  4. Two Stage Least Squares Method
  5. System Method

6. Generalised Least Square
7. Seemingly Unrelated Regression Equations

**Rank and Order Conditions for Identification**

Order condition:

$$K - k \geqslant m - 1 \tag{360}$$

Rank condition: =>

$$\rho(A) \geqslant (M-1)(M-1) \tag{361}$$

order of the matrix.

M = number of endogenous variables in the model

K = number of exogenous variables in the model including the intercept

m = number of endogenous variable in an equation

k = number of exogenous variables in a given equation

Rank condition is defined by the rank of the matrix, which should have a dimension (M-1), where M is the number of endogenous variables in the model.

Determining the Rank of the Matrix

- 1. Rank of matrix is the order of non-singular matrix
  2. Rank matrix is formed from the coefficients of the variables (both endogenous and exogenous) excluded from that particular equation but included in the other equations in the model.
  3. The rank condition tells us whether the equation under consideration is identified or not.
  4. The order condition tells us if it is exactly identified or overidentified.

Steps for Rank Condition

- 1. Write down the system in the tabular form
  2. Strike out the coefficients corresponding to the equation to be identified
  3. Strike out the columns corresponding to those coefficients in 2 which are nonzero.
  4. The entries left in the table will give only the coefficients of the variables included in the system but not in the equation under consideration. From these coefficients form all possible A matrices of order M-1 and obtain a corresponding determinant. If at least one of these determinants is non-zero then that equation is identified.

Summary of Order and Rank Conditions of Identification

- 1. If $(K-k) > (m-1)$ and the order of rank $\rho(A)$ is M-1 then the concerned equation is overidentified.
  2. If $(K-k) = (m-1)$ and the order of rank $\rho(A)$ is M-1 then the equation is exactly identified.
  3. If $(K-k) \geqslant (m-1)$ and he order of rank $\rho(A)$ is less than M-1 then the equation is underidentified.
  4. If $(K-k) < (m-1)$ the structural equation is unidentified. The the order of rank $\rho(A)$ is less M-1 in this case.

### 6.8.3 Empirical Part: Procedure in PcGive

- 1. construct data set in macroeconomic variables ( Y, C, I , G, T , X, M, MS, i, inflation, wage rate, exchange rate etc)
  2. save data in *.csv format
  3. Start GiveWin and PcGive and open data file
  4. choose multiple equation dynamic modelling
  5. determine endogenous and exogenous variables and run simultaneous equation using 3SLS or FIML
  6. Study coefficients
  7. Change policy variables and construct few scenarios

Homework: construct reasonable small scale macro model from the data in macro.csv. Project values of exogenous variables; do forecasts.

### 6.8.4 Excercise 9

1. Suppose that you have a simple model of consumption and income as following
Consumption function:

$$C_t = \beta_0 + \beta_1 Y_t + u_t \tag{362}$$

National income identity:

$$Y_t = C_t + I_t \tag{363}$$

1. (a) Use rank and order conditions to find whether the consumption function is identified in this model.
   (b) Write a reduced form for this system. Show how you could retrieve the structural coefficients$\beta_0$ and $\beta_1$ if you applied OLS to this reduced form.
   (c) Show that application of OLS to (1) generates a biased estimate of $\beta_1$.
   (d) What other method would you recommend to get an unbiased and best estimator for this model? Write steps to be followed until you get the structural coefficients $\beta_0$ and $\beta_1$.
   (e) Write a short note on how this model could be used to make a historical simulation of consumption and income series.

2. Consider a market model for a particular product.
Demand: $Q_t^d = \alpha_0 + \alpha_1 P_t + \alpha_2 I_t + u_{1,t}$ (1)
Supply: $Q_t^s = \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + u_{2,t}$ (2)
Here $Q_t^d$ is quantity demanded and $Q_t^s$ is quantity supplied, $P_t$ is the price of commodity,$P_{t-1}$ is price lagged by one period, $I_t$ is income of an individual, $u_{1,t}$ and $u_{2,t}$ are independently and identically distributed (iid) error terms with a zero mean and a constant variance.$Q_t$ and $P_t$ are endogenous variables and $P_{t-1}$and $I_t$ are exogenous variables $\alpha_0$,$\alpha_1$,$\alpha_2$,and $\beta_0$,$\beta_1$,$\beta_2$are six parameters defining the system.

1. (a) How can simultaneity bias occur if one tries to apply OLS to each of the above equations.
   (b) Use rank and order conditions to judge whether each of these two equations are over-, under- or exactly identified.
   (c) Write down the reduced form for this system.
   (d) How would you estimate the coefficients of the reduced form equations? Write down the estimator.
   (e) If equations are identified explain how you may retrieve the structural parameters $\alpha_0$,$\alpha_1$,$\alpha_2$,and $\beta_0$,$\beta_1$,$\beta_2$, and from the coefficients of the reduced form equations.

### 6.8.5 AR, MA, ARMA and ARIMA Forecasting

**AR(1) forecast**

$$y_t = \delta + \theta_1 y_{t-1} + e_t \tag{364}$$

h =1 ahead Forecast

$$y_{T+1} = \delta + \theta_1 y_T + e_{T+1} \qquad e_{T+1} \tilde{} N(0,1) \tag{365}$$

Mean forecast:

$$\widehat{y}_{T+1} = E\left(y_{T+1}\right) = \delta + \theta_1 y_T \tag{366}$$

Estimate of Forecast error

$$\widehat{e}_{T+1} = y_{T+1} - \widehat{y}_{T+1} = \delta + \theta_1 y_T + e_{T+1} - \delta - \theta_1 \widehat{y}_T \tag{367}$$

Variance of h =1 Forecast error

$$var\left(\widehat{e}_{T+1}\right) = \sigma_e^2 \tag{368}$$

h =2 ahead Forecast

$$y_{T+2} = \delta + \theta_1 y_{T+1} + e_{T+1} \qquad e_{T+2} \tilde{} N(0,1) \tag{369}$$

Mean forecast:

$$\widehat{y}_{T+2} = E\left(y_{T+2}\right) = \delta + \theta_1 y_{T+1} \tag{370}$$

Estimate of Forecast error

$$\begin{aligned} \widehat{e}_{T+2} &= y_{T+2} - \widehat{y}_{T+2} = \delta + \theta_1 y_{T+1} + e_{T+2} - \delta - \theta_1 \widehat{y}_{T+1} \\ &= e_{T+2} + \theta_1\left(y_{T+1} - \widehat{y}_{T+1}\right) = e_{T+2} + \theta_1 e_{T+1} \end{aligned} \tag{371}$$

Variance of Forecast error

$$var\left(\widehat{e}_{T+2}\right) = \sigma_e^2\left(1 + \theta_1^2\right) \tag{372}$$

h period ahead Forecast

$$y_{T+h} = \delta + \theta_1 y_{T+h-1} + e_{T+h} \qquad e_{T+h} \tilde{} N(0,1) \tag{373}$$

Mean forecast:

$$\widehat{y}_{T+h} = E\left(y_{T+h}\right) = \delta + \theta_1 \widehat{y}_{T+h-1} \tag{374}$$

Estimate of Forecast error

$$\begin{aligned} \widehat{e}_{T+h} &= y_{T+h} - \widehat{y}_{T+2} = \delta + \theta_1 y_{T+h-1} + e_{T+h} - \delta - \theta_1 \widehat{y}_{T+h-1} \\ &= e_{T+h} + \theta_1 \left(y_{T+h-1} - \widehat{y}_{T+h-1}\right) = e_{T+h} + \theta_1 e_{T+h-1} \end{aligned} \tag{375}$$

Variance of Forecast error

$$var\left(\widehat{e}_{T+h}\right) = \sigma_e^2 \left(1 + \theta_1^2 + \theta_1^2 + ... + \theta_1^{2(h-1)}\right) \tag{376}$$

**MA(1) forecast** Forecast with MA(1)

$$y_t = \mu + e_t + \alpha_1 e_{t-1} \tag{377}$$

h=1 period ahead forecast

$$y_{T+1} = \mu + e_{T+1} + \alpha_1 e_T \tag{378}$$

Mean forecast

$$E\left(y_{T+1}\right) = \widehat{y}_{T+1} = \mu + \alpha_1 e_T \tag{379}$$

Forecast error

$$y_{T+1} - \widehat{y}_{T+1} = \mu + e_{T+1} + \alpha_1 e_T - \mu - \alpha_1 e_{T+1} = e_{T+1} \tag{380}$$

Variance of forecast:

$$var\left(y_{T+1} - \widehat{y}_{T+1}\right) = var\left(e_{T+1}\right) = \sigma_e^2 \tag{381}$$

h=2 period ahead Forecast

$$y_{T+2} = \mu + e_{T+2} + \alpha_1 e_{T+1} \tag{382}$$

Mean forecast

$$E\left(y_{T+2}\right) = \widehat{y}_{T+2} = \mu \tag{383}$$

Forecast error

$$y_{T+2} - \widehat{y}_{T+2} = \mu + e_{T+2} + \alpha_1 e_{T+1} - \mu = e_{T+2} + \alpha_1 e_{T+1} \tag{384}$$

$$var\left(y_{T+2} - \widehat{y}_{T+2}\right) = var\left(e_{T+2}\right) = var\left(e_{T+2} + \alpha_1 e_{T+1}\right) = \sigma_e^2 \left(1 + \alpha_1^2\right) \tag{385}$$

Similarly mean and variance of h period ahead forecast:

$$y_{_{T+h}} = \mu + e_{_{T+h}} + \alpha_1 e_{_{T+h-1}} \tag{386}$$

$$E\left(y_{T+h}\right) = \widehat{y}_{_{T+h}} = \mu \tag{387}$$

Forecast error

$$y_{_{T+h}} - \widehat{y}_{_{T+h}} = \mu + e_{_{T+h}} + \alpha_1 e_{_{T+h-1}} - \mu = e_{_{T+h}} + \alpha_1 e_{_{T+h-1}} \tag{388}$$

$$var\left(y_{_{T+2}} - \widehat{y}_{_{T+2}}\right) = var\left(e_{T+h}\right) = var\left(e_{_{T+h}} + \alpha_1 e_{_{T+h-1}}\right) = \sigma_e^2\left(1 + \alpha_1^2\right) \tag{389}$$

**ARMA(1,1) forecast** Forecasts using ARMA(1,1) process:

$$y_t = \delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1} \tag{390}$$

h=1 period ahead Forecast

$$y_{_{T+1}} = \delta + \theta_1 y_{t-1} + e_{_{T+1}} + \alpha_1 e_{_T} \tag{391}$$

Mean forecast

$$E\left(y_{_{T+1}}\right) = \widehat{y}_{_{T+1}} = \delta + \theta_1 y_{t-1} + \alpha_1 e_{_T} \tag{392}$$

Forecast error

$$\begin{aligned} \widehat{e}_{_{T+1}} &= \left(y_{_{T+h}} - \widehat{y}_{_{T+h}}\right) = \\ \delta + \theta_1 y_{_{t-1}} + e_{_{T+1}} + e_{_{T+1}} + \alpha_1 e_{_T} - \delta - \theta_1 y_{t-1} - \alpha_1 e_{_T} &= e_{_{T+1}} \end{aligned} \tag{393}$$

Forecast error

$$\begin{aligned} \widehat{e}_{_{T+1}} &= \left(y_{_{T+h}} - \widehat{y}_{_{T+h}}\right) = \delta + \theta_1 y_{_{t-1}} + e_{_{T+1}} + e_{_{T+1}} \\ +\alpha_1 e_{_T} - \delta - \theta_1 y_{t-1} - \alpha_1 e_{_T} &= e_{_{T+1}} \end{aligned} \tag{394}$$

Variance of Forecast error

$$var\left(\widehat{e}_{_{T+1}}\right) = var\left(y_{_{T+h}} - \widehat{y}_{_{T+h}}\right) = var\left(e_{_{T+1}}\right) = \sigma_e^2 \tag{395}$$

$$y_t = \delta + \theta_1 y_{t-1} + e_t + \alpha_1 e_{t-1} \tag{396}$$

h=2 period ahead Forecast

$$y_{_{T+2}} = \delta + \theta_1 y_{t+1} + e_{_{T+2}} + \alpha_1 e_{_{T+1}} \tag{397}$$

Mean forecast and Forecast error

$$E\left(y_{T+2}\right)=\widehat{y}_{_{T+2}}=\delta+\theta_1 y_{t+1} \tag{398}$$

$$\begin{aligned}\widehat{e}_{_{T+2}} &= \left(y_{_{T+2}}-\widehat{y}_{_{T+2}}\right)=\delta+\theta_1 y_{_{t+1}}+e_{_{T+2}}+\alpha_1 e_{_{T+1}}-\delta-\theta_1\widehat{y}_{T+1}\\ &= \theta_1\left(y_{_{t+1}}-\widehat{y}_{T+1}\right)+e_{_{T+2}}+\alpha_1 e_{_{T+1}}=\left(\theta_1+\alpha_1\right)e_{_{T+1}}+e_{_{T+2}}\end{aligned} \tag{399}$$

Variance of Forecast error

$$var\left(\widehat{e}_{_{T+1}}\right)=var\left[\left(\theta_1+\alpha_1\right)e_{_{T+1}}+e_{_{T+2}}\right]=var\left(e_{_{T+1}}\right)=\sigma_e^2\left[\left(\theta_1+\alpha_1\right)^2+1\right] \tag{400}$$

h=3 period ahead Forecast

$$y_{_{T+2}}=\delta+\theta_1 y_{t+2}+e_{_{T+3}}+\alpha_1 e_{_{T+2}} \tag{401}$$

Mean forecast

$$E\left(y_{T+3}\right)=\widehat{y}_{_{T+3}}=\delta+\theta_1\widehat{y}_{t+2} \tag{402}$$

Forecast error and Variance of Forecast error

$$\begin{aligned}\widehat{e}_{_{T+3}} &= \left(y_{_{T+3}}-\widehat{y}_{_{T+3}}\right)=\delta+\theta_1 y_{_{t+2}}+e_{_{T+3}}+\alpha_1 e_{_{T+2}}-\delta-\theta_1\widehat{y}_{T+2}\\ &= \theta_1\left(y_{_{t+2}}-\widehat{y}_{T+2}\right)+e_{_{T+3}}+\alpha_1 e_{_{T+2}}\\ &= e_{_{T+3}}+\alpha_1 e_{_{T+2}}+\left(\theta_1+\alpha_1\right)e_{_{T+2}}+e_{_{T+2}}\end{aligned} \tag{403}$$

$$\begin{aligned}var\left(\widehat{e}_{T+3}\right) &= var\left[e_{_{T+3}}+\alpha_1 e_{_{T+2}}+\left(\theta_1+\alpha_1\right)e_{_{T+2}}+e_{_{T+2}}\right]\\ &= \sigma_e^2\left[1+\left(1+\alpha_1\right)^2+\left(\theta_1+\alpha_1\right)^2\right]\end{aligned} \tag{404}$$

## 6.9 VAR analysis

Consider a vector autoregressive model of order 2, VAR(2) given below.

$$y_t=a_{10}+a_{11}y_{t-1}+a_{12}y_{t-2}+b_{11}x_{t-1}+b_{12}x_{t-2}+e_{1,t} \tag{405}$$

$$x_t=a_{20}+a_{21}y_{t-1}+a_{22}y_{t-2}+b_{21}x_{t-1}+b_{22}x_{t-2}+e_{2,t} \tag{406}$$

where and are two variables for time t range from 1 ... $T$ periods. Errors of each equation, $e_1$and $e_2$, are identically and independently distributed with zero mean and constant variance and covariance between and is assumed zero.

a. Evaluate the relationship between and in the long run.

Answer: Long run relationship is obtained by imposing the steady state relations:

$$\overline{y}=a_{10}+a_{11}\overline{y}+a_{12}\overline{y}+b_{11}\overline{x}+b_{12}\overline{x} \tag{407}$$

$$\overline{y} = \frac{a_{10}}{1 - a_{11} - a_{12}} + \frac{(b_{11} + b_{12})}{1 - a_{11} - a_{12}}\overline{x} \tag{408}$$

$$\overline{x} = a_{20} + a_{21}\overline{y} + a_{22}\overline{y} + b_{21}\overline{x} + b_{22}\overline{x} \tag{409}$$

$$\overline{x} = \frac{a_{20}}{1 - b_{21} - b_{22}} + \frac{(a_{21} + a_{22})}{1 - b_{21} - b_{22}}\overline{y} \tag{410}$$

b. Provide impulse response analysis for and of a unit shock in $e_{1,t}$ and $e_{2,t}$.
Use lag operator $y_{t-1} = Ly_t; y_{t-2} = Ly_{t-1} = L^2 y_t$;Then the system changes to

$$y_t = a_{10} + a_{11}Ly_t + a_{12}L^2 y_t + b_{11}Lx_t + b_{12}L^2 x_t + e_{1,t} \tag{411}$$

$$x_t = a_{20} + a_{21}Ly_t + a_{22}L^2 y_t + b_{21}Lx_t + b_{22}L^2 x_t + e_{2,t} \tag{412}$$

$$y_t = \frac{a_{10}}{1 - a_{11}L - a_{12}L^2} + \frac{(b_{11} + b_{12})}{1 - a_{11}L - a_{12}L^2}x_t + \frac{1}{1 - a_{11}L - a_{12}L^2}e_{1,t} \tag{413}$$

$$x_t = \frac{a_{10}}{1 - b_{11}L - b_{12}L^2} + \frac{(a_{11} + a_{12})}{1 - b_{11}L - b_{12}L^2}y_t + \frac{1}{1 - b_{11}L - b_{12}L^2}e_{2,t} \tag{414}$$

Terms $\frac{1}{1-a_{11}L-a_{12}L^2}e_{1,t}$ and $\frac{1}{1-b_{11}L-b_{12}L^2}e_{2,t}$ give the impulse response of the first and second equations respectively.

c. Indicate and explain criteria to determine the order of a VAR model like this:

It is wise to use from general to specific approach of David Hendry to determine the order of VAR . First start the model with a large number of lags and then keep reducing the number of lags until the significant relation is found. Likelihood ratio tests are suggested for this.

d. What extra information is needed to make a h period ahead forecast using the above model? VAR is a time series model. Given the past values of time series, it requires distribution of the error terms for h period ahead forecasts.

e. A diagram can show how the variance of the forecast error and the confidence interval of a forecast are sensitive to the number of periods in the forecast horizon. The confidence level of forecast increases with the larger horizon of the forecasts.

## 6.10 ARCH /GARCH Modelling of Volatility

Introduction to ARCH

OLS estimates are based on the normality of errors, which are assumed to have constant mean and variance. Engel (1983) argued that many economic time series go through a series of ups and downs. Upward trend continues up to a significant length of time. and so does the downward trend.

As such the conditional mean and variance of these series are not constant. Modelling mean and variance of series simultaneously is the essence of the autoregressive conditional heteroskedasticity (ARCH) model.

The variance of error term is persistent and shown by autoregressive process of variances.

This technique has been widely used to measure the volatility of financial time series such as the interest rate, inflation, stock prices, returns to assets, growth rates, trends in trades.

Bollersleve (1987) modified it to generalised autoregressive conditional heteroskedasticity (GARCH) models.

How ARCH and GARCH models are used to test the heteroskedasticity are discussed first followed by illustrations on variants of them used to study the clustering of heteroskedastic errors commonly used in the literature.

Engel (1983) autoregressive conditional heteroskedasticy (ARCH): more useful for time series data

Model $Y_t = \beta_0 + \beta_1 X_{1,t} + \beta_2 X_{2,t} + \beta_3 X_{3,t} + .... + \beta_k X_{k,t} + e_t$

$\varepsilon_t \tilde{} N\left(0, \left(\alpha_0 + \alpha_2 e_{t-1}^2\right)\right)$

$$\sigma_t^2 = \alpha_0 + \alpha_2 e_{t-1}^2 \tag{415}$$

Here $\sigma_t^2$ not observed. Simple way to estimate this is to run OLS of $Y_t$ and get $\widehat{e}_t^2$. Then assume an ARCH (1) of errors as

$\widehat{e}_t^2 = \alpha_0 + \alpha_2 \widehat{e}_{t-1}^2 + v_t$ or ARCH (p) $\widehat{e}_t^2 = \alpha_0 + \alpha_2 \widehat{e}_{t-1}^2 + \alpha_3 \widehat{e}_{1-1}^2 + \alpha_4 \widehat{e}_{1-1}^2 + .. + \alpha_p \widehat{e}_{1-p}^2 + v_t$

Compute the test statistics

$n.R^2 \sim \chi_{df}^2$

Again if the calculated $\chi_{df}^2$ is greater than table value there is an evidence of ARCH effect and heteroskedasticity.

Economies are characterised by turbulent high volatility periods followed by quite and peaceful low volatility periods.

Decision makers require some estimates of expected values as well as volatility to reflect on the uncertainties causes by such phenomenon.

Recently stock prices rose continuously from 2002 to mid 2008 and then fell sharply in 2008 and 2009 and can be expected to rise in the next few years. Billions are lost and won because of volatilities in these series.

Engle (1987) proposes modelling expected value and volatility simultaneously by ARCH using iterative Maximum Likelihood procedure as:

$$Y_t = \beta_0 + \beta_1 X_{1,t} + e_t \tag{416}$$

where $e_t \tilde{} N\left(0, \sigma_t^2\right) = N\left(0, h_t\right)$; $h_t = \sigma_t^2$.

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 \tag{417}$$

Bollerslev (1987) generalised autoregressive conditional heteroskedasticy (GARCH) process is more general. For instance GARCH (1,1). Mean and variance equations take the following form:

$$Y_t = \beta_0 + \beta_1 X_{1,t} + e_t \tag{418}$$

$$\sigma_t^2 = \alpha_0 + \alpha_2 \widehat{e}_{t-1}^2 + \beta \sigma_{t-1}^2 + v_t \tag{419}$$

GARCH (p,q)

$\sigma_t^2 = \alpha_0 + \alpha_2 \widehat{e}_{t-1}^2 + \alpha_3 \widehat{e}_{t-2}^2 + \alpha_4 \widehat{e}_{t-3}^2 + .. + \alpha_p \widehat{e}_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 + ..\beta_q \sigma_{t-q}^2 + .. + v_t$

Compute the test statistics $n.R^2 \tilde{} \chi_{df}^2$

Sometimes written as

$h_t = \alpha_0 + \alpha_2 \widehat{e}_{t-1}^2 + \alpha_3 \widehat{e}_{t-2}^2 + \alpha_4 \widehat{e}_{t-3}^2 + .. + \alpha_p \widehat{e}_{t-p}^2 + \beta_1 h_{t-1} + \beta_2 h_{t-2} + ..\beta_q h_{t-q} + .. + v_t$

*where* $h_t = \sigma_t^2$

### 6.10.1 Homework

1. Select one time series such as stock price or quarterly consumption. Estimate AR, MA, ARMA models to make ten period ahead forecasts.

2. Estimate a VAR(2) of growth rate of DPD and inflation rate. Do impulse response analysis using unit shocks to ten period horizon. Provide the forecast and the confidence interval of forecast.

3. Take daily stock price of a certain company. Fit appropriate ARCH/GARCH models to explain volatility. Make a forecast.

## 6.11 Panel Data Models

Many economic issues require cause-effect analyses of cross-sections of individuals, households or countries over time. The major issue that economists like to know remains whether coefficients vary across individual observations at a particular time or whether variables have any systematic pattern over time. For instance macroeconomists are interested to know what makes growth rates differ across countries at a particular year and of the same country over time. In other words they want to find out whether there are any country specific and time specific effects on economic growth? Panel studies of growth studies are carried out to know the determinants of growth of an individual country or a group of countries over time. Similarly microeconomic studies aim to investigate whether wages and earnings linked to characteristic of workers and other environmental factors over time or whether profits vary systematically by firms and by production periods.

When confronted with these questions an appropriate econometric method requires using all observations across individuals for each time period under investigation. The paned data regression models in econometrics takes account of all cross section and time series observations. The major emphasis lies on decomposing total variation within a group and between the various groups. Subscript i, t refer to individual and time period respectively

Panel Data: Fixed Effects

$$y_{i,t} = \alpha_i + x_{i,t}\beta + e_{i,t} \qquad e_{i,t} \sim IID\left(0, \sigma_e^2\right) \tag{420}$$

where parameter $\alpha_i$ picks up the fixed effects that differ among individuals but constant over time,$\beta$ is the vector of coefficients on explanatory variables. These parameters can be estimated by OLS when $N$ is small but not when that is large.

The model need to be transformed to the least square dummy variable method when $N$ is too large. For this take time average

$$\overline{y}_i = \alpha_i + \overline{x}_i\beta + e_i \qquad \overline{y}_i = T^{-1}\sum_i y_{i,t} \tag{421}$$

Take the mean difference

$$y_{i,t} - \overline{y}_i = \left(x_{i,t} - \overline{x}_i\right)\beta + \left(e_{i,t} - e_i\right) \tag{422}$$

fixed effect least square dummy variable estimator of $\beta$ is

$$\beta_{FE} = \left( \sum_{t}^{T} \sum_{i}^{N} (x_{i,t} - \overline{x}_i) (x_{i,t} - \overline{x}_i)' \right)^{-1} \sum_{t}^{T} \sum_{i}^{N} (x_{i,t} - \overline{x}_i) (y_{i,t} - \overline{y}_i)' \tag{423}$$

$$\alpha_i = \overline{y}_i - \overline{x}_i \beta_{FE} \tag{424}$$

These estimators are unbiased, consistent and efficient with corresponding covariance matrix given by:

$$cov(\beta_{FE}) = \sigma_e^2 \left( \sum_{t}^{T} \sum_{i}^{N} (x_{i,t} - \overline{x}_i) (x_{i,t} - \overline{x}_i)' \right)^{-1} \tag{425}$$

where $\sigma_e^2 = \frac{1}{N(T-1)} \sum_{t}^{T} \sum_{i}^{N} (y_{i,t} - \alpha_i - x_{i,t}\beta_{FE})$

Random effect models are more appropriate for analysing determinants of growth as

$$y_{i,t} = \mu + x_{i,t}\beta + \alpha_i + e_{i,t} \tag{426}$$

where $\alpha_i \tilde{} \; IID\left(0, \sigma_\alpha^2\right)$ are individual specific random errors and $e_{i,t} \tilde{} \; IID\left(0, \sigma_e^2\right)$ are remaining random errors.

$$\alpha_i \iota_T + e_i \qquad where \; \iota_T = (1, 1, .....1) \tag{427}$$

$$var(\alpha_i \iota_T + e_i) = \Omega = \sigma_\alpha^2 \iota_T \iota_T' + \sigma_e^2 I_T \tag{428}$$

Errors are correlated therefore this requires estimation by the Generalised Least Square estimator. Transform the model by pre-multiplying by $\Omega^{-1}$ where $\Omega^{-1} = \sigma_e^2 \left[ I_T - \frac{\sigma_\alpha^2}{\sigma_e^2 + T\sigma_\alpha^2} \iota_T \iota_T' \right]$

Panel Data Model: Random Effect

$$\begin{aligned} \beta_{GLS} \quad = \quad & \left( \sum_{t}^{T} \sum_{i}^{N} (x_{i,t} - \overline{x}_i) (x_{i,t} - \overline{x}_i)' + \psi T \sum_{i}^{N} (x_{i,t} - \overline{x}_i) (x_{i,t} - \overline{x}_i)' \right)^{-1} \\ & \left( \sum_{t}^{T} \sum_{i}^{N} (x_{i,t} - \overline{x}_i) (y_{i,t} - \overline{y}_i)' + \psi T \sum_{i}^{N} (x_{i,t} - \overline{x}_i) (y_{i,t} - \overline{y}_i)' \right) \end{aligned} \tag{429}$$

### 6.11.1 Pooling Cross Section and Time Series: Seemingly Unrelated Regression (SUR) MODEL

- SUR if formed by stacking models

$$Y_1 = X_1 \beta + e_1 \tag{430}$$

$$Y_2 = X_2 \beta + e_2 \tag{431}$$

$$... \tag{432}$$

$$Y_m = X_m\beta + e_m \tag{433}$$

There are m equations and T observations in the SURE system (in growth rate example we have 151 countries and 31 observations). They can be stacked into one large equation system as following.

Pooling Cross Section and Time Series: Seemingly Unrelated Regression (SUR) MODEL

There are m equations and T observations in the SURE system (in growth rate example we have 151 countries and 31 observations). They can be stacked into one large equation system as following.

$$\begin{bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_m \end{bmatrix} = \begin{bmatrix} X_1 & 0 & . & . & 0 \\ 0 & X_2 & . & . & 0 \\ . & . & X_3 & . & 0 \\ . & . & . & . & . \\ 0 & 0 & . & . & X_m \end{bmatrix} \begin{matrix} \beta_1 \\ \beta_2 \\ . \\ . \\ \beta_m \end{matrix} + \begin{bmatrix} e_1 \\ e_2 \\ . \\ . \\ e_m \end{bmatrix} \tag{434}$$

- Each $Y_m$ and $e_m$has a dimension of T by 1 and $X_m$ has T by K dimension and each $\beta_m$ has K by 1 dimension. The covariance matrix of errors has TM by TM dimension.

Pooling Cross Section and Time Series: Seemingly Unrelated Regression (SUR) MODEL

Aitken generalised least square

$$\widehat{\beta} = \left[X'V^{-1}X\right]^{-1} X'V^{-1}Y = \left[X'\left(\Sigma^{-1}\otimes I\right)X\right]^{-1} X'\left(\Sigma^{-1}\otimes I\right)Y \tag{435}$$

$$\widehat{\beta} = \begin{bmatrix} \sigma_{1,1}X_1'X_1 & \sigma_{1,1}X_1'X_2 & \sigma_{1,1}X_1'X_3 & \sigma_{1,m}X_1'X_m \\ \sigma_{2,1}X_2'X_1 & \sigma_{2,2}X_2'X_2 & \sigma_{2,3}X_2'X_3 & \sigma_{2,m}X_2'X_m \\ & & & \\ \sigma_{m,1}X_m'X_1 & \sigma_{m,2}X_m'X_2 & \sigma_{m,3}X_m'X_3 & \sigma_{m,4}X_m'X_m \end{bmatrix} \begin{bmatrix} \sum \sigma_{1,j}X_1'Y_j \\ \\ \\ \sum \sigma_{m,j}X_m'Y_j \end{bmatrix} \tag{436}$$

Steps for SUR Estimation

- Estimate each equation separately using the least square technique.
- Use the least square residuals from step 1 to estimate the error term.
- Use the estimates from the second step to estimate two equations jointly within a generalised least square framework. If m=2 the variance covariance matrix will be as given below.

Estimation of Seemingly Unrelated Regression (SUR) by GLS

- $$\Omega = \begin{pmatrix} \sigma_1^2 & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_2^2 \end{pmatrix} \tag{437}$$

  Using a theorem in matrix algebra W can be decomposed into two parts as

$$P'P = \Omega^{-1} \tag{438}$$

Use this partition of $\Omega$ to transform the original model as

$$Y = X\beta + \varepsilon \tag{439}$$

$$\beta_{OLS} = (X'X)^{-1}(X'Y) \tag{440}$$

Estimation of Seemingly Unrelated Regression (SUR) by GLS
Transform it to

$$P'Y = P'X\beta + P'\varepsilon \tag{441}$$

$$Y^* = X^*\beta + \varepsilon^* \tag{442}$$

$$\beta_{GLS} = (X'P'PX)^{-1}(X'P'PY) \tag{443}$$

In matrix notation

$$\beta_{GLS} = \left(X^{*\prime}\Omega^{-1}X^*\right)^{-1}\left(X^{*\prime}\Omega^{-1}Y^*\right) \tag{444}$$

$\Omega^{-1}$ is inverse of variance covariance matrix.

The GLS estimates are best, linear and unbiased estimators of the coefficients in the SURE system.

Total Effect

$$\beta_{OLS} = \frac{\sum_t^T \sum_i^N \left(X_{i,t} - \overline{X}\right)\left(Y_{i,t} - \overline{Y}\right)}{\sum_t^T \sum_i^N \left(X_{i,t} - \overline{X}\right)\left(X_{i,t} - \overline{X}\right)} = \frac{t_{x,y}}{t_{x,x}} \tag{445}$$

$$t_{x,y} = \sum_t^T \sum_i^N \left(X_{i,t} - \overline{X}_i + \overline{X}_i - \overline{X}\right)\left(Y_{i,t} - \overline{Y}_i + \overline{Y}_i - \overline{Y}\right) \tag{446}$$

$$\begin{aligned} t_{x,y} &= \sum_t^T \sum_i^N \left(X_{i,t} - \overline{X}_i\right)\left(Y_{i,t} - \overline{Y}_i\right) + T\sum_i^N \left(\overline{X}_i - \overline{X}\right)\left(\overline{Y}_i - \overline{Y}\right) \\ &= W_{x,y} + b_{x,y} \end{aligned} \tag{447}$$

Within and between effects

Between group effect

$$\beta_b = \frac{b_{x,y}}{b_{x,x}} = \frac{\sum_t^T \left(X_{i,t} - \overline{X}_i\right)\left(Y_{i,t} - \overline{Y}_i\right)}{\sum_t^T \left(X_{i,t} - \overline{X}_i\right)^2} \tag{448}$$

Within group effect

$$\beta_W = \frac{W_{x,y}}{W_{x,x}} = \frac{\sum_i^N \left(X_{i,t} - \overline{X_t}\right)\left(Y_{i,t} - \overline{Y_t}\right)}{\sum_t^T \left(X_{i,t} - \overline{X_t}\right)^2} \tag{449}$$

$$\beta_{OLS} t_{x,x} = t_{x,y} = W_{x,y} + b_{x,y} = \beta_W \frac{W_{x,x}}{W_{x,y} + b_{x,y}} + \beta_b \frac{b_{x,x}}{W_{x,y} + b_{x,y}} \tag{450}$$

### 6.11.2 Panel Unit root test

Increasingly recent studies have looked into nonstationarity and heterogeneity issues in panel data model. Levin and Lin (1992)

$$\Delta y_{i,t} = \alpha_i + \rho y_{i,t-1} + \sum_{k=1}^{n} \phi_k \Delta y_{i,t-1} + \delta_i t + \theta_t + u_{i,t} \tag{451}$$

$H_0 : \rho = 1$ against $H_0 : \rho < 1$

Levin, A., C. Lin and C. Chu (2002): "Unit Root Tests in Panel Data: Asymptotic and finite sample properties", Journal of Econometrics, 108, p.12-24.

IM, Pesharan and Shin (1997)

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$$\Delta y_{i,t} = \alpha_i + \rho y_{i,t-1} + \sum_{k=1}^{n} \phi_i \Delta y_{i,t-1} + \delta_i t + \theta_t + u_{i,t} \tag{452}$$

Heterogeneity in unit roots: against no unit root

$$t_{IPS} = \frac{\sqrt{N}\left(t - \frac{1}{N}\sum_{k=1}^{n} E\left[t_{iT} | \rho_i = 0\right]\right)}{\sqrt{\frac{1}{N}\sum_{k=1}^{n} var\left[t_{iT} | \rho_i = 0\right]}} \Longrightarrow N(0,1) \tag{453}$$

Maddala and Wu (1999) tests for unbalanced panel

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## 6.12 Linear probability, Probit and logit models

- Alternative names: dichotomous dependent variables, discrete dependent random variable, binary variable, either or choice variables

$$Y_i = \int \begin{array}{l} Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i \quad \text{if the event occurs} \\ 0 = \text{otherwise} \end{array} \tag{454}$$

Examples

- the labour force participation (1 if a person participates in the labour force, 0 otherwise)
- yes or no vote in particular issue ; to marry or not to marry; to study further or to start a job
- to buy or not to buy a particular stock
- choice of transportation mode to work (1 if a person drives to work, 0 otherwise)
- Union membership (1 if one is a member of the union, 0 otherwise)
- Owning a house (1 if one owns 0 otherwise)
- Multinomial choices: work as a teacher, or as a clerk, or as a self employed or professional or as a factory worker
- Multinomial ordered choices: strongly agree, agree, neutral, disagree

Linear Probability Model

$$Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i \tag{455}$$

where $Y_i = 1$ if person owns a house, 0 otherwise; $X_i$ is family income.
$E\left[\left(Y_i = 1\right)/X_i\right]$ probability that the event y will occur given x

$$E\left[\left(Y_i = 1\right)/X_i\right] = 0 \times \left[1 - P_i\right] + 1 \times P_i = P_i \tag{456}$$

$$0 \leqslant E\left[\left(Y_i = 1\right)/X_i\right] = P_i = \beta_1 + \beta_2 X_i \leqslant 1 \tag{457}$$

- Problem: Errors are heteroskedastic.

$$\varepsilon_i = 1 - \beta_1 - \beta_2 X_i \quad with \quad (1 - P_i) \tag{458}$$

$$\varepsilon_i = -\beta_1 - \beta_2 X_i \quad with \quad P_i \tag{459}$$

Variance of error in a linear probability model

$$var\left(\varepsilon_i\right) = \left(1 - \beta_1 - \beta_2 X_i\right)^2 \ \left(1 - P_i\right) + \left(-\beta_1 - \beta_2 X_i\ \right)^2 P_i \tag{460}$$

$$\sigma^2 = \left(1 - \beta_1 - \beta_2 X_i\right)^2 \ \left(-\beta_1 - \beta_2 X_i\right) + \left(-\beta_1 - \beta_2 X_i\ \right)^2 \left(1 - \beta_1 - \beta_2 X_i\right) \tag{461}$$

$$\sigma^2 = \left(1 - \beta_1 - \beta_2 X_i\right)\left(\beta_1 + \beta_2 X_i\right) \ = \left(1 - P_i\right) \ P_i \tag{462}$$

Variance depends on X.
Limitations of a linear probability model
It is possible to transform this model to make it homeskedastic by dividing the original variables by

$$\sqrt{\left(1 - \beta_1 - \beta_2 X_i\right)\left(\beta_1 + \beta_2 X_i\right)} \ = \sqrt{\left(1 - P_i\right) \ P_i} = \sqrt{W_i} \tag{463}$$

$$\frac{Y_i}{\sqrt{W_i}} = \frac{\beta_1}{\sqrt{W_i}} + \beta_2 \frac{X_i}{\sqrt{W_i}} + \frac{\varepsilon_i}{\sqrt{W_i}} \tag{464}$$

- It does not guarantee that the probability lies inside (0,1) bands
- Probability in non-linear phenomenon: at very low level of income a family does not own a house; at very high level of income every one owns a house ; marginal effect of income is very negligible. The linear probability model does not explain this fact well.

Probit Model

- $$\begin{aligned} \Pr(Y_i = 1) &= \Pr(Z_i^* \leq Z_i) = F(Z_i) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_i} e^{-\frac{t^e}{2}} dt \\ &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\beta_1 + \beta_2 X_i + \varepsilon_i} e^{-\frac{t^e}{2}} dt \end{aligned} \tag{465}$$

- Here $t$ is standardised normal variable, $t \tilde{} N(0,1)$

  probability depends upon unobserved utility index $Z_i$ which depends upon observable variables such as income. There is a thresh-hold of this index when after which family starts owning a house, $Z_i \geqslant Z_i^*$

Logit Model

- variable $Y_i$ which takes value 1 ($Y_i = 1$) if a student gets a first class mark, value 0 ($Y_i = 0$)otherwise.
- Probability of getting a first class mark in an exam is a function of student effort index denoted by $.Z_i$ ; where $P_i = \frac{1}{1+e^{-Z_i}}$

  $Z_i = \beta_1 + \beta_2 X_i + \varepsilon_i$ An example of a logit model: what determines that a student gets the first class degree?

$$Z_i = \beta_1 + \beta_2 H_i + \beta_3 E_i + \beta_4 A_i + \beta_2)_i + \varepsilon_i \tag{466}$$

  H = hours of study, E= exercises, A = attendance in lectures and classes; P = papers written for assignment.

- Ratio of odds: $\frac{P_i}{1-P_i} = \frac{1+e^{Z_i}}{1+e^{-Z_i}} = e^{Z_i}$ ;taking log of the odds $ln\left(\frac{P_i}{1-P_i}\right) = Z_i$

Features of a logit Model

- – probability goes from 0 to 1 as the index variable goes from -$\infty$ to +$\infty$. Probability lies between 0 and 1.
  – Log of the odds is linear in x, characteristic variables but probabilities themselves are not linear but non linear function of the parameters. Probabilities are estimated using the maximum likelihood method.

- Any explanatory variable that determines the value of $Z_i$ , measures how the log of odds of an event (i.e. owning a house) changes as a result of change in explanatory variable such as income.
- We can calculate $P_i$ for given estimates of $\beta_1$and $\beta_2$or all other .$\beta_i$
- Limiting case when $P_i$ =1; $ln\left(\frac{P_i}{1-1}\right)$ or when $P_i$ =0 ; $ln\left(\frac{0}{1-0}\right)$OLS cannot be applied in such case but the maximum likelihood method may be used to estimate the parameters.

Tobit Model

- It is an extension of the probit model, named after Tobin. We observe variables if the event occurs: ie if some one buys a house. We do not observe explanatory variables for people who have not bought a house. The observed sample is censored, contains observations for only those who buy the house.

$$Y_i = \int \begin{array}{l} Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i \quad \text{if the event occurs} \\ 0 = \text{otherwise} \end{array} \tag{467}$$

  - is equal to is the event is observed equal to zero if the event is not observed.
  - It is unscientific to estimate probability only with observed sample without worrying about the remaining observations in the truncated distribution. The Tobit model tries to correct this bias.
  - Inverse Mill's ratio: Example first estimate probability of work then estimate the hourly wage as a function of socioeconomic background variables

Summary of Probability Models
The effect of observed variables on probability

- –

$$\frac{\partial P_i}{\partial x_{i,j}} = \left\{ \begin{array}{l} \beta_j \\ \beta_j P_j \left(1 - P_j\right) \\ \beta_j \phi \left(Z_i\right) \end{array} \right. \tag{468}$$

  - where $Z_i$ =$\beta_0 + \sum_{i=1}^{k} \beta_i X_{i,j}$ and $\phi$ is the standard normal density function.

## 6.13 Bayesian Statistics and Econometrics

- Analysis in classical econometrics is based on an assumption that true population parameters are constant but sample estimates of those parameters are random variables distributed normally around those population parameters.
- Bayesian regard true parameter to be a random variable, priors are updated frequently upon arrival of new data.

In Bayesian analysis the value of true parameter $\theta$ unknown like in the classical approach but it is not fixed.

Instead $\theta$ has a probability distribution and it is updated continuously based on sample information – priors.

The prior density is given by $f(\theta)$ and this may represent all available information up to that point.

$$f(\theta_1) = \int_{-\infty}^{\infty} f(\theta)\, d\theta_2 d\theta_3 ... d\theta_n \tag{469}$$

The sample density of variable y is treated as conditional on the random variable $\theta$ given by $f(y/\theta)$

### 6.13.1 Bayesian Rule

Let $p(A, B)$ be the joint probability of occurring events A and B together, $p(B)$ be the marginal probability of B without any respect to occurrence of A then the probability of A conditional on the occurrence of B is

$$p(A|B) = \frac{p(A, B)}{p(B)} \tag{470}$$

Similarly the probability of B conditional on the occurrence of A is

$$p(B|A) = \frac{p(A, B)}{p(A)} \tag{471}$$

Substituting the value of $p(A, B)$ from 1 the probability of $B$ conditional on the occurrence of $A$ is

$$p(B|A) = \frac{p(A|B)\, p(B)}{p(A)} \tag{472}$$

Bayesian econometrics is application of Bayesian Rule repeatedly for estimation on unknown parameters.

For data $y$ and parameter $\theta$ then by replacing $A$ by y and $B$ by $\theta$

$$p(\theta|y) = \frac{p(y|\theta)\, p(\theta)}{p(y)} \tag{473}$$

Since $p(y)$ does not involve $\theta$ it can be ignored and this function written as

$$p(\theta|y) \propto p(y|\theta)\, p(\theta) \tag{474}$$

where $p(\theta|y)$ is the posterior density, $p(y|\theta)$ is the likelihood function and $p(\theta)$ is the prior density. Posterior thus is proportional to likelihood times prior. Posterior combines both data and non-data information.

See more in http://www.bayesian.org/

# 7 Game Theory: Introduction

- Economic activities of consumers, producers, governments and nations or regions are interdependent.
- Game theory provides tools to study the strategic interactions among such economic agents where decisions taken by one individual depend on actions taken by others.
- Each game has a number of players who choose a set of strategies and rules. .Optimal choices available to one depend on choices made by others.
- Pay-offs are clearly defined for each player strategy pairs.
- Strategic modelling like this started with classics such as Cournot (1838), Bertrand (1883), Edgeworth (1925) von Neumann and Morgenstern (1944), Nash (1950). It is developing very fast in recent years following works of Kuhn (1953), Shapley (1953),Shelten ( 1965) Aumann (1966) Scarf (1967), Shapley and Shubic (1969), Harsanyi(1967), Spence (1974), Kreps (1990), Fundenberg and Tirole (1991) and Binmore (1992).

Elements of a Game

- Rational Players
- Strategies
- Payoff matrix

Table 21: Structure of a Game

| | | Player A | |
|---|---|---|---|
| | | Strategy 1 | Strategy 2 |
| Player B | Strategy 1 | $\left(\Pi^R_{1,1}, \Pi^C_{1,1,}\right)$ | $\left(\Pi^R_{1,2}, \Pi^C_{1,2}\right)$ |
| | Strategy 2 | $\left(\Pi^R_{2,1}, \Pi^C_{2,1,}\right)$ | $\left(\Pi^R_{2,2}, \Pi^C_{2,2}\right)$ |

$\Pi^R_{1,1}$ is pay-off to row player if he plays strategy 1 and the column player plays strategy 1.

Players like to maximise their own pay-off given opponent's strategy; B will choose strategy 1 or 2 that maximises his/her payoff looking at the choice of player A. Most games have equilibrium from which players do not have any incentive to move away.

### 7.0.2 Solution concepts of GAMES

1. minmax = maxmin solutions (applies TPZS games)
2. Dominant strategy
3. Mixed strategy
4. Nash equilibrium (non cooperative games)
5. Repeated games

6. Mechanism design.

Non-Cooperative Game (prisoner's dilemma)
Finding Nash solution (underscore the best strategy to a player i given the choice of the opponent.

Table 22: Prisoners' Dilemma Game

| | | Player A | |
|---|---|---|---|
| | | Confess | Dont Confess |
| Player B | Confess | $(\underline{-5}, \underline{-5})$ | $(\underline{-1}, -10)$ |
| | Dont Confess | $(-10, \underline{-1})$ | $(-2, -2)$ |

Nash Equilibrium: Prisoner's Dilemma

- Fact: both players did a crime together. Police suspects and arrest both of them.
- Playing non cooperatively each convicts another. Game results in Nash solution (confess, Confess) = $(-5, -5)$; Each ends up with 5 years in prison.
- By confessing, each gives evidence to the police to determine the highest possible punishment.
- If they had cooperated remaining silent, police would not have enough evidence.
- Each would have been given only two years of prison $(-2, -2)$. This is Pareto optimal outcome, "where no one could be made better off without making someone worse-off".
- Cooperation is better but each think that other player will cheat and therefore doesn't cooperate. Therefore stay longer in jail.
- There are many example of prisoner's dilemma game in real world -pricing and output in a cartel, pollution, tax-revenue.

### 7.0.3 Bargaining Game

- The very common example for bargaining game is splitting a pie between two individuals.
- The sum of the shares of the pie claimed by both cannot exceed more than 1, otherwise each will get zero.
- If we denote these shares by $\theta_i$ and $\theta_j$ then $\theta_i + \theta_j \leq 1$ is required for a meaningful solution of the game where each get $\theta_i \geq 0$ and $\theta_j \geq 0$ payoff. When $\theta_i + \theta_j > 1$ then and $\theta_i = 0$ and $\theta_j = 0$ .
- Standard technique to solve this problem is to use the concept of Nash Product.
- Solution is to divide the pie equally between two. No other solution is stable.

### 7.0.4 Principal-Agent Game

**Role of Signalling** Asymmetric information in Used Car Market -Akerlof's Model of Asymmetric Information

- Sellers know exactly quality of cars but buyers do not.
- Equilibrium is affected when sellers have more information than buyers.
- Market has plums: good cars and lemons: bad cars
- Seller knows his quality of cars but buyers do not
- Market for good cars disappear because of existence of bad cars in the market.
- Demand for high quality car falls and demand for low quality cars rise.
- Ultimately only low quality cars remain in the market.

Asymmetric information in Used Car Market -Signalling solves the Problem

- signals: warranty and Guarantee
- Providing warranty less costly for high quality cars as they last long.
- Warranty is costly for low quality cars as they frequently break down.
- Buyers can decide whether a car is good or bad looking at the warranty and pay appropriately.
- Right signalling can remove inefficiency due to incomplete information.
- Markets for both types of car can operate efficiently by right signals of warranty and Guarantee

Pooling, Separating and Mixed Equilibrium

- Complete market failure

pooling equilibrium (same price for good and bad cars; good cars disappear from the market)

- Complete market success

Separating equilibrium where players act as they should according to the signal (prices according to quality)

- Partial market success

(both good and bad cars are bought, some feel cheated)
Near Market failure (mixed strategies) Bayesian updating mechanism at work

Education Level- A Signal of Productive Worker

- An employer does not know is more productive and who is less productive
- It pays the same wage rate to both productive and unproductive workers

- market is inefficient, it drives out more productive workers.
- Workers can signal their quality by the level of educational attainment, then market may work well.
- Less costlier for high quality worker to get education.
- costlier for low quality worker to get the specified education.
- so the low quality worker gets no education, but the higher quality worker gets education.
- Employers pay according to the level of education.
- Education works as a signalling device and makes the market efficient.
- Education separates the equilibrium.

## 7.1 Mechanism design

### 7.1.1 An Example of price discrimination by a low cost airlines

- Economy and Business Class Ticket Problem for Airlines (Based on Dixit et. al. (2009))
- Two types of travellers: economy and business
- Assume 100 travellers and 70 of them economy type tourists and 30 business type first class.

| | Cost of | Reservation Price | | Airline's Profit | |
|---|---|---|---|---|---|
| | the Airlines | Tourists | Business | Tourists | Business |
| Economy | 100 | 140 | 225 | 40 | 125 |
| First Class | 150 | 175 | 300 | 25 | 150 |

- Economy class tickets cost less than the business class.
- Business traveller is ready to pay higher price than economy class for both economy and first class but the airlines cannot separate them out.

Why Mechanism Design for Price Discrimination: Low Cost Airlines Example

- Economy class tickets cost less than the business class.
- Business traveller is ready to pay higher price than economy class for both economy and first class but the airlines cannot separate them out.
- Business traveller may well buy economy class ticket rather then business class.
- Airlines likes to build a mechanism so that business class buy business class tickets and economy class buy economy class ticket.
- What is the profit to the airlines if it knows reservation prices of tourists and business group of travellers?
- How would this profit change in business type buy the economy class ticket?

- What is the incentive compatible price that the airlines can offer to the business group?

Incentive Compatible Mechanism

What would happen if the split between the business and economy class is 50/50? What will be the optimal reaction of the airlines?

Profit in an ideal scenario ( perfect price discrimination; if the airlines knew each customer type)

$$\begin{aligned} 30(300-150)+(140-100)(70) &= 30\times 150+40\times 70 \\ &= 4500+2800=7300 \end{aligned} \tag{475}$$

Business travellers have consumer surplus of 225 -140 = 85 in economy class ticket. For this all 30 of may decide to buy economy class ticket. Then the profit of the airlines when the airlines fails to screen customers will be

$$(140-100)(100)=4000 \tag{476}$$

Airlines should give consumer surplus of 85 to business traveller and charge them (300-85) = 215. This will alter their profit

$$\begin{aligned} 30(215-150)+(140-100)(70) &= 30\times 65+40\times 70 \\ &= 1950+2800=4750 \end{aligned} \tag{477}$$

Incentive Compatible and Participation Constraints

- Airline initially does not have enough information on types of customers
- It should design incentive compatible pricing scheme so that business class travellers do not defect to economy class.
- This requirement is contained in the incentive compatible constraint. If it charges 240 for the business class then the their consumer surplus will be equal (300-240) = 60 from business class travel and (225-165)=60
- However 140 is the maximum the tourist class traveller is ready to pay. If the airline raises price to 165 they will lose all tourist travellers. Mechanism requires fulfillment of the participation constraint.
- Airlines should operate taking account of the participation constraint of tourists and incentive compatible constraint of the business travellers.
- X $<$ 140 is the participation constraint; incentive compatible constraint is 225 -X $<$ 300-Y or Y $<$ X+75

Charging 215 for the business class and 140 for the economy class is the solution to the mechanism design problem

Mechanism when the composition of travellers change

- Suppose the composition of travellers changes to 50% of each. Profit with the above price mechanism

$$\begin{aligned} 50(215-150)+(140-100)(50) &= 50\times 65+40\times 50 \\ &= 3250+2000=5250 \end{aligned} \tag{478}$$

- It is more profitable to scrap the tourist class tickets instead and charge the business class its full reservation price

$$50(300-150)=50\times 150=7500 \tag{479}$$

- There are relatively few customers but all are willing to pay higher price. There is no problem of screening as the airlines now does not serve to the tourist class at all.

### 7.1.2 Mechanism to ensure high efforts by a CEO

- Owners of a company are concerned about a project that would earn them 600,000 if successful.
- Probability of success with normal effort from the manager is 60 percent and this can increase up to 80 percent if the manager puts extra efforts.
- The basic salary of the manager is 100,000. He would put extra efforts only if he is paid additional amount of at least 50,000. Owners cannot monitor whether the manager is putting high or low efforts.

a) Is it profitable to pay extra for the manager?
Profit without paying extra: 0.6 * 600,000 - 100,000 = 260, 000
Profit with extra incentive payment: 0.8 * 600,000 - 150,000 = 330, 000
Extra payment can make up to 70,000 with probability of 0.8.
Once extra payment is made how can owners make sure that he puts extra efforts? This requires evaluation of incentive compatibility and participation constraints.
Mechanism to ensure high efforts by a CEO
a) Incentive compatibility constraint

$$(s+0.8b)-(s+0.6b)\geqslant 50,000 \tag{480}$$

$$0.2b\geqslant 50,000 \tag{481}$$

b = 250,000
b) Participation constraint:

$$(s+0.8b)\geqslant 150,000 \tag{482}$$

$$s=150,000-0.8b;\quad s=150,000-0.8\,(250,000)=-50,000 \tag{483}$$

It is not possible to hire manager with negative salary. At most managers can be conditioned to bonus payment but with zero salary.
Mechanism to ensure high efforts by a CEO

$$(0 + 0.8b) \geqslant 150,000 \tag{484}$$

$$200,000 \geqslant 150,000 \tag{485}$$

Pay 200,000 and the manager will put maximum effort.
c) Is it profitable to pay extra 200,000 as an incentive payment?
Profit with incentive payment
0.8 * 600,000 - 200,000 = 280, 000
Profit without incentive payment
0.6 * 600,000 - 100,000 = 260, 000
Thus profit increases by 20,000 with the incentive payments.

### 7.1.3 Mechanism design in renting lands

Proposition 1: Results of fixed fee contract and joint profit maximisation are equivalent
Proposition 2: Hire contract is incentive incompatible and leads to production inefficiency
Proposition 3: Moral hazard problem and production inefficiency exists in revenue sharing contingent contract
Proposition 4: Profit sharing contract is efficient and free of moral hazard problem
Price and cost

$$P = 24 - 0.5q \qquad C = 12q \tag{486}$$

Revenue

$$R = P.q \tag{487}$$

Mechanism design in renting lands
Under the joint profit maximisation agreement

$$\Pi(q) = P.q - C = (24 - 0.5q)\, q - 12q = 24q - 0.5q^2 - 12q \tag{488}$$

Under the fixed fee contract tenant maximises

$$\Pi(q) = P.q - C - F = (24 - 0.5q)\, q - 12q - F = 24q - 0.5q^2 - 12q - F \tag{489}$$

Under both these arrangements

$$\Pi'(q) = 24 - q - 12 = 0 \tag{490}$$

$$q = 12; \quad p = 18; \; R = 216; \quad C = 144; \; \Pi(q) = 72 \tag{491}$$

Mechanism design in renting lands

- 72 is the total profit. It is divided between the tenant and the landlord by their mutually agreed arrangement. Under the fixed fee contract landlord may fix the amount that he needs at 48.

- Then the residual 24 profit goes to the tenant.
- This arrangement achieves production efficiency, is incentive compatible, fulfils the participation constraint and motivates to put the optimal effort and solves the moral hazard problem.

Hire contract

- Landowner can hire workers in fixed fee basis, say 12 per unit of output a.
- This does not motivate tenant to work because his cost per a is also 12 and so does not make any profit. Landlord has to raise payment to tenant to say 14 to motivate him to work.
- Then the profit maximisation problem of the landlord will be

$$\Pi(q) = P.q - C = (24 - 0.5q)\, q - 14q = 24q - 0.5q^2 - 14q \tag{492}$$

$$\Pi'(q) = 24 - q - 14 = 0 \tag{493}$$

$$q = 10; \quad p = 19; \; R = 190; \quad C = 120; \, \Pi_{LL}(q) = 50; \Pi_T(q) = 20 \tag{494}$$

The tenant has incentive to overproduce whenever is paid more than 12.
Revenue sharing contract

- Let the landlord enter into a revenue sharing contract whereby she gets $\frac{1}{4}$ th of the revenue and leavening $\frac{3}{4}$ of revenue to the tenant who also bears all production cost. The profit function of the tenant is now modified as

$$\Pi(q) = \frac{3}{4} P.q - C = \frac{3}{4}(24 - 0.5q)\, q - 12q \tag{495}$$

$$\Pi'(q) = 6 - \frac{3}{4} q = 0 \tag{496}$$

$$\begin{aligned} q &= 8; \quad p = 20; \; R = 160; \quad C = 96; \, \Pi_{LL}(q) = \frac{3}{4}(160) \\ &= 120; \Pi_T(q) = \frac{1}{4}(160) = 40 \end{aligned} \tag{497}$$

Profit of tenant = 120 - 96 =24
This level of production is not incentive compatible for the land-lord who would be interested in maximising revenue by producing 24.
Profit sharing contract

- Now let us assume the landlords and tenants enter into a profit sharing deal, say 1/3rd of profit goes to the tenant and 2/3rd to the landlord.

$$\frac{1}{3}\Pi(q) = \frac{1}{3}(P.q - C) = \frac{1}{3}\left(24q - 0.5q^2 - 12q\right) \tag{498}$$

$$\Pi'(q) = 4 - \frac{1}{3}q = 0 \tag{499}$$

$$\begin{aligned} q &= 12; \quad p = 18; \; R = 216; \quad C = 144; \, \Pi(q) = 72; \\ \Pi_{LL}(q) &= 48; \Pi_T(q) = 24 \end{aligned} \tag{500}$$

There are many other situations, including optimal tax designs, optimal price discrimination, fund management, management of theme-park, renting of buildings, collection of taxes or tariffs, union-management contracts, where these types of models have been applied.

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# A general equilibrium model for analysis of fiscal policy

(Based on Bhattarai K. (2006) Macroeconomic Impacts of Taxes: A General Equilibrium Analysis, Indian Economic Journal, 54:2:95-116, July-Sept.)

## 7.2 Introduction

The multiplier analysis of changes in government expenditure, investment or exports in the presence of lump sum or proportional income taxes under the fixed price ISLM model, as commonly found in the usual macro-textbooks, is not sufficient for analysis of impacts of taxes in an economy. Taxes have wide ranging impacts both in supply and demand sides of the markets for goods and services and factors of production and they distort commodity prices, wage rates which significantly affect the allocation of resources in the economy. Reduction of taxes on commodities not only shifts the aggregate demand towards right by increasing the disposable income of the households but also causes an increase in spending along the aggregate demand line as lower prices induce more demand by households with accompanying income and substitution effects. A reduction in the labour income tax rate lowers the cost of production of the firms and raises the demand for labour.

It also raises labour supply in response to higher net of tax wage rate. More production reduces prices of commodities from the supply side and raises the real income of households who supply labour to a firm. Higher real income may induce further increase in the demand for leisure and reduce the supply of labour. An increase in both direct and indirect tax rates have similar effect on the opposite direction.
Overall macroeconomic effects of taxes involve very complicated substitution and income effects at micro level, which should not be missed in a formal analysis of the fiscal policy. The incidence of tax may fall either on consumer or producer though ultimately consumers pay the gross of tax prices prevailing in the market including the wedge above the net of tax prices that is actually received by the firms. Similarly firms pay gross of tax wage rates to its workers but can recover the amount of taxes they paid by charging higher prices on commodities to the households. It is the net of tax wage rate that really matters when a household makes labour-leisure and consumption-saving decisions. Absence of evaluation of these direct and indirect impacts of taxes in discussion of expansionary or contractionary fiscal or monetary policy is a serious shortcoming of the traditional ISLM models built in tradition of Keynes (1936) and Hicks (1937).

There is a long standing debate on relative merits and demerits of the direct and indirect taxes in terms of their incidence and impact to the tax payers. Microeconomic analysis based on Marshallian or Hicksian analysis of demand and supply takes a partial equilibrium view which may be true from the point of a single firm or the consumer but not sufficient from the perspective of the economy as a whole. A partial equilibrium approach to analyse economy wide problem often generates misleading results. The public finance literature as in Ramsey (1927), Pigou(1947), Atkinson and Stiglitz (1976 ), Mirlees (1971), Whalley (1975), Boadway and Bruce (1992), Peroni (1995), Hutton and Ruocco (1999) Iwamoto and Shibata (1999) Seidman and Lewis (1999) make this point to some extent but very few of these studies investigate the macroeconomic impacts and miss explicit general equilibrium solution of the tax induced macroeconomic impacts on output and employment as clearly as desirable (Rebelo(1991)). Most of these analyses focus on impact on welfare of individuals or redistribution impacts of transfers (Pechman (1987), Atkinson and Stiglitz (1976)).

The simplest possible general equilibrium model is often presented for a Crusoe type economy taking interaction between consumption and production by a representative household. Such a model can be used to evaluate the impact of both direct and indirect taxes on consumption, leisure, labour supply, output and employment. The welfare cost is measured in terms of reduction in the utility of a representative household, who supplies labour to the firm, by examining the optimality of income or consumption taxes separately or jointly under the general equilibrium framework in which the decisions of households to maximize utility subject to its budget constraint are linked to the decision of producers to maximize profit choosing the optimal units of employment through the real wage rate. The analytical solution of the model with no taxes is presented in section II, with taxes in section III, a numerical illustration for a hypothetical economy in section IV before applying the model to the actual data set for the placecountry-region economy in section V. Paper ends with conclusions followed by references.

II. Micro-founded general equilibrium model with a representative household and a representative firm. A simple general equilibrium model represents an economy in which a representative household maximises utility by consuming goods and services supplied by producers and paying for them by income that it receives in return of supply of labour and capital inputs to the producers. Firms optimise profit combining inputs with the existing technology in production and rewarding inputs according to its marginal productivity. Tax policies of government influence both production

and consumption sides of the economy by affecting prices of inputs and outputs. By distorting the marginal conditions of optimisation, these taxes influence choices of goods and services by households and use of inputs by producers. The incidence and impact of taxes on consumption may differ from taxes on labour income depending on the key parameters for share or elasticities of substitution in consumption or in the production sides of the economy.
A general equilibrium implies a set of prices that eliminate excess supply or excess demand and where these prices and wage rates are consistent with the preferences and endowments of households and technology of firms. The perfect match between demand and supply for both goods and services and inputs of production follow from the properties of utility and production functions as given by explicit analytical solutions in the next section.

Household's Problem

Consider an economy with a representative household and a representative firm. The household tries to maximise utility by consuming goods and services and enjoying leisure subject to its budget constraint. The producer wants to maximise profit by selling goods produced using the labour supplied by the household. The household maximisation problem can be stated in terms of preferences, budget and time constraints as the following:

$$Max \;\; U = c^{\phi} l^{1-\phi} \tag{501}$$

Subject to:

i. $l + h^s = 1$ time constraint

ii. $wh^s + \pi = pc$ budget constraint

iii. $c \geq 0; l \geq 0; h^s \geq 0$ non-negativity constraint.

where c is consumption, l is leisure and $h^s$ is labour supply, p is the price of the commodity, w is the wage rate; $\pi$is the profit from owning the firm, $\phi$ is weight of consumption and$(1-\phi)$weight of leisure in utility.

For simplicity, normalise labour endowment to 1, define leisure as $l = 1 - h^s$and substitute it into the utility function to get,

$$U = c^{\phi} \left(1 - h^s\right)^{1-\phi} \tag{502}$$

The Lagrangian function for the constrained optimisation is given by

$$L\left(c, l, \lambda\right) = c^{\phi} \left(1 - h^s\right)^{1-\phi} + \lambda \left[wh^s + \pi - pc\right] \tag{503}$$

It has three choice variables $(c, l, \lambda)$, $\lambda$is shadow price of income. The first order conditions (FOC) for utility maximisation are:

$$\frac{\partial L\left(c, l, \lambda\right)}{\partial c} = \phi c^{\phi-1} \left(1 - h^s\right)^{1-\phi} - \lambda p = 0 \tag{504}$$

$$\frac{\partial L\left(c, l, \lambda\right)}{\partial h^s} = \left(1 - \phi\right) c^{\phi} \left(1 - h^s\right)^{-\phi} (-1) + \lambda w = 0 \tag{505}$$

$$\frac{\partial L\left(c, l, \lambda\right)}{\partial \lambda} = wh^s + \pi - pc = 0 \tag{506}$$

Equation (3) and (4) give the marginal utility from consumption and leisure respectively. These three equations can be used to solve for three choice variables. Demand for consumption goods can be derived by dividing FOC (3) by FOC (4) and solving for c.

$$\frac{\frac{\partial L(c,l,\lambda)}{\partial h^s}}{\frac{\partial L(c,l,\lambda)}{\partial c}} = \frac{(1-\phi)\, c^{\phi} \left(1-h^s\right)^{-\phi} (-1)}{\phi c^{\phi-1} \left(1-h^s\right)^{1-\phi}} = \frac{w}{p} \tag{507}$$

This optimising condition implies that the marginal rate of substitution between leisure and consumption should equal the real wage rate in equilibrium. It is clear that the consumption demand depends on real wage rate and the work efforts.

$$c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p} \tag{508}$$

In the absence of taxes, the market clearing condition in this single good economy implies that the income of the household equals spending on consumption: $wh^s + \pi = pc$. Using (7) this budget constraint can be rewritten as:

$$h^s \frac{w}{p} + \frac{\pi}{p} = c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p} \tag{509}$$

Now (8) can be solved for the household's labour supply

$$h^s \frac{w}{p} + \frac{\pi}{p} = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p} \tag{510}$$

=>

$$h^s \left(1 + \frac{\phi}{1-\phi}\right)\frac{w}{p} = \left(\frac{\phi}{1-\phi}\right)\frac{w}{p} - \frac{\pi}{p} \tag{511}$$

$$h^s \left(\frac{1}{1-\phi}\right)\frac{w}{p} = \left(\frac{\phi}{1-\phi}\right)\frac{w}{p} - \frac{\pi}{p}\frac{1-\phi}{1-\phi} \tag{512}$$

$$h^s \frac{w}{p} = \phi\frac{w}{p} - \frac{\pi}{p}(1-\phi) \tag{513}$$

From the optimising behavior of a representative household the supply of labour, demand for leisure and consumption become functions of the real wage rate and profit.

$$h^s = \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}} \tag{514}$$

leisure

$$l = 1 - h^s = 1 - \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}} \tag{515}$$

$$c = \left(\frac{\phi}{1-\phi}\right)\left(1-h^s\right)\frac{w}{p} \tag{516}$$

=

$$c = \left(\frac{\phi}{1-\phi}\right)\left(1 - \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}}\right)\frac{w}{p} \tag{517}$$

The real wage rate that is consistent with optimal demand for labour and profit from the firm's side of optimisation is found in the next section.

## 7.3 Firm's Problem

The representative firm receives the net of tax price of goods by selling them to the households and pays the gross of tax wage rate for labour. Its maximisation problem, given the technology, input and output prices can be stated as:

$$Max \;\; \pi = py - wh^d \tag{518}$$

subject to :

i. $y \leq \left(h^d\right)^\alpha$ technology constraint ii. $y \geq 0; h^d \geq 0$ non negativity constraint; where y is the output supplied by the firm and $h^d$is its demand for labour. All above conditions can be collapsed into one objective function for the firm.

$$\pi = py - wh^d = p\left(h^d\right)^\alpha - wh^d \tag{519}$$

From the first order condition, firm's demand for labour input can be derived using the profit maximisation condition, which implies

$$\frac{\partial \pi}{\partial h^d} = p\alpha\left(h^d\right)^{\alpha-1} - w = 0 \tag{520}$$

=>

$$h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} \tag{521}$$

This optimal demand for labour by the firm in terms of the real wage rate can be substituted in the objective function of the firm (12) to express profit in terms of the real wage rate.

$$\begin{aligned} \frac{\pi}{p} &= y - w\frac{h^d}{p} = \left(h^d\right)^\alpha - \frac{w}{p}h^d \\ &= \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}} - \frac{w}{p}\left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = \left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right] \end{aligned} \tag{522}$$

### 7.3.1 Definition of a Competitive Equilibrium

In this model a competitive equilibrium is obtained by a real wage rate that guarantees the demand for labour by firms to be exactly equal to the supply of labour by households, the time endowments of households to be exactly equal to the sum of supply of labour and demand for leisure, and the production (supply) of output to be exactly equal to the demand for consumption by the household. Given the convexity of preferences and technology such equilibrium exists, is unique and stable.

The real wage rate can be determined by using the labour market clearing condition where the demand for labour by the firm equals the supply of labour by the household, $h^d = h^s$. That implies in equilibrium the demand for labour,

$$h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} \tag{523}$$

, should equal the supply of labour,

$$h^s = \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}} \tag{524}$$

. Then using the expression for the real profit from (15) this equality should fulfil the following identity.

$$h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = h^s = \frac{\phi\frac{w}{p} - \frac{\pi}{p}(1-\phi)}{\frac{w}{p}} = \frac{\phi\frac{w}{p} - \left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right](1-\phi)}{\frac{w}{p}} \tag{525}$$

Two sides of the labour market in (16) can be solved for equilibrium real wage rate as a function of preference and technology parameters of the model.

$$\frac{w}{p} = \frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]^{\alpha-1}} \tag{526}$$

The equilibrium quantities of l and $h^d$ are determined by this equilibrium wage rate:

$$\hat{h}^d = \hat{h}^s = \left(\frac{1}{\alpha}\frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]^{\alpha-1}}\right)^{\frac{1}{\alpha-1}} \tag{527}$$

the demand for leisure:

$$\hat{l} = 1 - \hat{h}^s = 1 - \left(\frac{1}{\alpha}\frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]^{\alpha-1}}\right)^{\frac{1}{\alpha-1}} \tag{528}$$

Given the real wage rate and labour supply and leisure, the amount of consumption (c) and the level of output supplied (y) in the no tax case can be obtained by substituting real wage rate from (17) into the demand for consumption function in (7) and labour demand function in (14)

$$\hat{c} = \left(\frac{\phi}{1-\phi}\right)\left(1 - \left(\frac{1}{\alpha}\frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]^{\alpha-1}}\right)^{\frac{1}{\alpha-1}}\right)\frac{\phi^{\alpha-1}}{\left[(1-\phi)\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]^{\alpha-1}} \tag{529}$$

Equilibrium output is then given by substituting the equilibrium labour input in the production function in (12)

$$\hat{y} = \left( \frac{1}{\alpha} \frac{\phi^{\alpha-1}}{\left[ (1-\phi) \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} + \phi \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \right]^{\alpha-1}} \right)^{\frac{\alpha}{\alpha-1}} \tag{530}$$

The representative household maximises utility using these optimal values of consumption and leisure in the utility function, $\hat{U} = \hat{c}^{\phi} \hat{l}^{1-\phi}$. This example explicitly shows how the price system allocates resources in an economy.

### 7.3.2 Micro-founded general equilibrium model with consumption and labour income taxes

When both consumption and incomes taxes are applied to raise the revenue the budget constraint changes to

$$w\left(1-t_l\right) h^s + \pi + R = p\left(1+t_c\right) c \tag{531}$$

, where $t_c$is tax rate on consumption and $t_l$ is the tax rate on labour income and R is the tax revenue which is given back to households as transfers. The household problem modifies but the firms problem remains the same as before as we have not considered taxes at the firm's profit.

$$Max \;\; U = c^{\phi} l^{1-\phi}$$

Subject to:

i. $l + h^s = 1$ time constraint

ii. $p\left(1+t_c\right) c = w\left(1-t_l\right) h^s + \pi + R$ budget constraint with taxes

iii. $pt_c c + wt_l h^s = R$ tax revenue and transfer

iv. $c \geq 0; l \geq 0; h^s \geq 0$ non-negativity constraint (1')

The optimisation problem in the tax case is similar but need to be take account of the tax distorted conditions. When the tax revenue function, R is substituted into the budget constraint, the tax budget constraint returns the no tax budget constraint.

$$p\left(1+t_c\right) c = w\left(1-t_l\right) h^s + \pi + R \tag{532}$$

(unknown char) $wh^s + \pi = pc$.

The term R can be treated as constant when the equal yield revenue constraint is imposed in the model otherwise it varies depending on the tax instruments and tax rates.

The optimal closed form solution of this model in the presence of tax requires tax-distorted first order conditions in the model which can be obtained using the Lagrangian function for constrained optimisation as:

$$L\left(c, l, \lambda\right) = c^{\phi} \left(1-h^s\right)^{1-\phi} + \lambda \left[ w\left(1-t_l\right) h^s + \pi + R - p\left(1+t_c\right) c \right] \tag{533}$$

The household has three choice variables $(c, l, \lambda)$, where $\lambda$is the shadow price of income and tax rates are among the choice variables in the optimal tax model. The first order necessary conditions

(FOC) give the marginal utility of consumption, and leisure or the disutility of work and the budget constraint as given below.

$$\frac{\partial L\left(c,l,\lambda\right)}{\partial c}=\phi c^{\phi-1}\left(1-h^{s}\right)^{1-\phi}-\lambda p\left(1+t_{c}\right)=0 \tag{534}$$

$$\frac{\partial L\left(c,l,\lambda\right)}{\partial h^{s}}=\left(1-\phi\right)c^{\phi}\left(1-h^{s}\right)^{-\phi}\left(-1\right)+\lambda w\left(1-t_{l}\right)=0 \tag{535}$$

$$\frac{\partial L\left(c,l,\lambda\right)}{\partial \lambda}=w\left(1-t_{l}\right)h^{s}+\pi+R-p\left(1+t_{c}\right)c=0 \tag{536}$$

or $wh^{s}+\pi=pc$

In equilibrium the marginal rate of substitution between consumption and leisure (ratio of FOC (4') to FOC (5')) equals the after tax real wage rate:

$$\frac{\frac{\partial L(c,l,\lambda)}{\partial h^{s}}}{\frac{\partial L(c,l,\lambda)}{\partial c}}=\frac{\left(1-\phi\right)c^{\phi}\left(1-h^{s}\right)^{-\phi}\left(-1\right)}{\phi c^{\phi-1}\left(1-h^{s}\right)^{1-\phi}}=\frac{w\left(1-t_{l}\right)}{p\left(1+t_{c}\right)} \tag{537}$$

Using results (4') to (7') the demand for consumption goods is obtained as a function of labour supply ( $h^{s}$) and the real wage rate ( $\frac{w}{p}$) and the consumption tax rate ( $t_{c}$) and labour income tax rate $t_{l}$.

$$c=\left(\frac{\phi}{1-\phi}\right)\left(1-h^{s}\right)\frac{w\left(1-t_{l}\right)}{p\left(1+t_{c}\right)} \tag{538}$$

The household's labour supply $h^{s}$ need to be solved as a function of $\frac{w}{p}$ and which can be done using equation

$$\frac{w}{p}h^{s}+\frac{\pi}{p}=c \tag{539}$$

.The counterpart of the equation (8) in the tax case is:

$$h^{s}\frac{w}{p}+\frac{\pi}{p}=c=\left(\frac{\phi}{1-\phi}\right)\left(1-h^{s}\right)\frac{w}{p}\left(\frac{1-t_{l}}{1+t_{c}}\right) \tag{540}$$

The equation (9') is derived after cancelling the tax revenue and transfer components of the tax distorted budget constraint as given in equation (2'). Collecting the terms of the household labour supply:

$$h^{s}\frac{w}{p}+\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_{l}}{1+t_{c}}\right)h^{s}=c=\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_{l}}{1+t_{c}}\right)-\left(\frac{\pi}{p}\right) \tag{541}$$

$$h^{s}\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_{l}}{1+t_{c}}\right)+1\right)=c=\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_{l}}{1+t_{c}}\right)-\left(\frac{\pi}{p}\right) \tag{542}$$

$$h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\frac{\pi}{p}\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)} \tag{543}$$

Using the value of optimal profit

$$h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)} \tag{544}$$

The definition of a competitive equilibrium does not change much in the tax distorted economy compared to no tax economy except that the real wage rate now reflects the tax distorted choices and equilibrium. The demand for labour by firms are reduced, the supply of labour by households may rise or fall though the labour market clearing conditions still holds implying the sum of the labour supply and leisure demand to be equal to the total time endowment of the households. The labour market equilibrium should be consistent to the goods market clearing condition whereby the output of the firm equals consumption demand by the household. In addition the government spending should equal tax revenue, which is met in this model by transferring all collected revenue to the household. Using these definitions of a competitive economy, the tax equilibrium can be determined by the real wage rate that equates the demand for and supply of labour in the tax distorted economy.

$$h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = h^s = \frac{\left(\frac{\phi}{1-\phi}\right)\frac{w}{p}\left(\frac{1-t_l}{1+t_c}\right) - \left(\left(\frac{w}{p}\right)^{\frac{\alpha}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\right)}{\frac{w}{p}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)} \tag{545}$$

$$\left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) = \left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right) - \left(\left(\frac{w}{p}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\right) \tag{546}$$

$$\left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) + \left(\left(\frac{w}{p}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right]\right) = \left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right) \tag{547}$$

$$\left(\frac{w}{p}\right)^{\frac{1}{\alpha-1}}\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) + \left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right] = \left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right) \tag{548}$$

$$\left(\frac{w}{p}\right)^{\frac{1}{\alpha-1}} = \frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) + \left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]} \tag{549}$$

Thus the real wage rate in equilibrium is given by

$$\frac{w}{p} = \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right) + \left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}\right]^{\alpha-1} \tag{550}$$

Once we have this real wage rate, the amount of labour supplied and demanded in the tax distorted economy can be evaluated using the labour demand equation in (14). Firms pay the gross of tax wage rate but households receive the net of tax wage. Households pay gross of tax prices for goods and services and firm's receive net of tax prices. The differences between these prices accrue to the government as the tax revenue. It cause distortions though the government ploughs back all of the revenues to households directly as transfers or as public goods or services.

From the above analysis it is obvious that the commodity and labour income taxes create a wedge between the producer prices received by firms and market prices paid by households in the goods market. The labour income tax puts a wedge between the gross of tax wage rate paid by firms and net of tax wage rate received by the households. These wedges may differ between these two tax instruments and may have different impacts on consumption, labour supply, output and demand for leisure.

$$h^d = \left(\frac{1}{\alpha}\frac{w}{p}\right)^{\frac{1}{\alpha-1}} \tag{551}$$

(unknown char)

$$h^s = h^d = \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}\right] \tag{552}$$

The labour supply in equations (20') is expressed also in terms of the real wage rate by using the equilibrium condition in the labour market, where demand for labour by the firms is consistent with the supply of the labour by the household. The labour supply then determines the level of output as:

$$\hat{y} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}\right]^{\alpha} \tag{553}$$

Leisure demand is then given by the difference between the endowment and labour supply.

$$\hat{l} = 1 - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}\right] \tag{554}$$

Consumption demand is obtained using the leisure in (22') above in (8').

$$\hat{c} = \left(\frac{\phi}{1-\phi}\right)\left[1 - \left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}} \left[\frac{\left(\frac{\phi}{1-\phi}\right)\left(\frac{1-t_l}{1+t_c}\right)}{\left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\left(\frac{\phi}{1-\phi}\left(\frac{1-t_l}{1+t_c}\right)+1\right)+\left\{\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}-\left(\frac{1}{\alpha}\right)^{\frac{1}{\alpha-1}}\right\}\right]}\right]\right]\frac{w\left(1-t_l\right)}{p\left(1+t_c\right)} \tag{555}$$

The level of utility to the representative household in the tax distorted economy is given by $\hat{U} = \hat{c}^{\phi}\hat{l}^{1-\phi}$.

As can be seen from the above derivation the price and real wage rates are the basis of all consumption and production decisions. Production occurs because there is demand for products by the households. Firms demand factors to produce output. They pay labour according to its marginal product. The remuneration for labour services provides income to the households, which they use to purchase consumption goods. Once the equilibrium price is determined that determines all other quantities, such as the demand for goods and leisure and the supply of labour by households and the demand for labour and supply of commodities by firms. Both labour and capital income taxes distort the equilibrium quantities and prices and have discernible impacts in allocation of resources in the economy.

### 7.3.3 A numerical example for the general equilibrium tax model

A simple numerical example is provided here for the above general equilibrium tax model and used it to measure the relative impacts of consumption and income taxes in the economy. These impacts vary according to the preferences of households in relation to technology of production available to firms. If households strongly prefer leisure rather than consumption, the costs of income taxes are likely to be higher than those of the consumption taxes. On the other hand if the households have stronger preferences for consumption than for leisure then consumption taxes might be costlier. As stated above these preference and technology factors jointly determine the prices and prices influence allocation of resources in the economy. Numerical values of model parameters are given in Table 1.

Two types of simulations are conducted using this model. The first one is the base case scenario constructed using a reasonable set of preference and technology parameters as given in Table 1. Then there is a no tax scenario where all taxes are eliminated, followed by scenarios with taxes either only on consumption or only on labour income. The impacts of tax policy experiments are determined by comparing the utility and changes in output and employment before and after the change in taxes.

In addition sensitivity analyses are conducted to see how the welfare and macroeconomic impacts change in response to a distribution of households by preference and that of firms by the production technology. The grids of parameters for sensitivity analyses are in Table 2. Again impacts of taxes are measured in terms of levels and changes in utility, output, leisure, labour supply and consumption.

### 7.3.4 Analysis of model results

The benchmark economy includes consumption tax rate of 17 percent and income tax rate of 35 percent, similar to the one that actually exists in many of the OECD countries including the UK In all experiments the government returns tax revenue to the household in the form of a lump sum transfer. The model is then used to evaluate the impacts of four different tax reform experiments: (1) the distortionary cost of both income and consumption taxes; (2) impact of a complete switch to the labour income tax holding the revenue fixed; (3) impact of a complete switch towards the consumption tax; and (4) the test of reliability and robustness of the model by examining the sensitivity to the key parameters of the model.

The Hicksian equivalent variations (EV) are presented in terms of the money metric utility in the counter factual scenario in comparison to the benchmark scenario by asking how much

the household benefits from the tax changes equivalent in terms of the original equilibrium. The corresponding compensating variation is also in terms of the money metric utility measuring the amount of compensation a consumer needs to bring back her to the original level of utility after the changes in tax rates.

The overall welfare costs of taxes, as presented in Table 3 above, show that the costs of using both consumption and income taxes are higher than of using only either the consumption or only the labour income tax. The overall distortionary impacts of both consumption and labour income taxes are up to 3.2 percent of the benchmark utility. This compares to results as contained in Bhattarai and Whalley (1999, 2003). If the revenue is returned as a lump sum form to the households, the model results confirm that the labour income tax has highly distortionary impact in the economy. It may cost up to 6.2 percent of the benchmark utility. Higher rate of labour income tax first reduces labour supply and output and consumption consequently. In comparison, sole reliance on only consumption taxes significantly lowers distortions than labour income taxes. Model calculations suggest that the cost of moving towards only consumption taxes is 0.05 percent of the benchmark utility. Thus the overall costs are lower when tax is only on consumption than when taxes are both on consumption and labour income simultaneously.

For this hypothetical economy taxes affect the aggregate output, employment, leisure, labour supply as well as consumption of the household as shown in Table 4 to Table 7. It is obvious that the adverse macroeconomic impacts of only labour income taxes are a lot higher than those of only consumption taxes.

When both labour income and consumption taxes are removed, households lose the amount of transfer income from the government but still it has a very good positive impact on output, consumption and utility level of the representative household. In contrast the labour income tax discourages labour supply relative to both the no tax and consumption tax only cases leading to highly distortionary effects on the economy.

This model generates predictable results when subject to sensitivity tests along the various rates of consumption and labour income tax rates. It confirms that the welfare costs of taxes rise proportionately to the squares of tax rates as suggested by the famous Harberger triangle, a measure of the dead weight loss of taxes.

The model also behaves well when subject to changes in the endowments. Households receive higher utility as their endowments rise but at a decreasing rate given the law of diminishing marginal utility. Since the consumer values both consumption and leisure, the increase in utility of increasing only consumption show a diminishing utility as does the increase in the share of labour in production which raises the marginal productivity of labour and reduces the amount of leisure in the utility function.

The welfare cost of taxes rises when tax rates increase as shown in Table 6. Changes in the base case utility are sensitive to time endowment of household (for instance in the case of a two member household compared to a single member household). The welfare levels are sensitive to the preferences and labour supply elasticity as shown in Table 7. The costs of a consumption tax are higher in an economy which prefers more leisure than consumption but are lower in an economy which prefers more consumption than leisure. This confirms to the diminishing marginal utility theory of consumption. Similarly labour bears the most burden of taxes when elasticity of labour demand is lower rather than when it is higher as shown in the last two columns in Table 7.

All above results show how the relative prices of commodities and labour determine the allocation of resources in an artificial economy. This model, however, can be applied to a real economy to assess the impacts of alternative tax measures on output, consumption, labour supply, leisure, price

of a commodity and wage rate in an economy. Though very simple, this model can answer a number of key policy questions that may confront the policy makers while considering the fiscal policy for an economy.

The model was applied taking actual data on endowments, consumption share and production share of the UK Economy for 2002. The consumption tax rate is set to 17 percent which is close enough to the standard VAT rate and the labour income tax rate is set to 35 percent. This is fairly representative tax rate in the UK when one considers the average of the base and higher income tax rates along with the national insurance payments that are applied to labour income in the UK. The labour endowment measured in terms of efficiency units, is set at 1000 billion which is close to the size of the economy of 2002. From the income distribution tables of the Office of National Statistics (2002) the share parameter (phi) in the consumption function is set to 0.6 and the elasticity of output to the labour input (alpha) set to 0.75. These values were confirmed by an econometric estimation.

The first scenario considers the efficiency impacts of removing all the taxes and transfers in the UK. When a representative household does not pay tax it also does not receive any transfer from the government. This is an extreme scenario in which all public services are provided by the private sector. The second scenario considers switching completely to the labour income tax and eliminating all indirect taxes. The third scenario, on the other hand is switching completely to consumption taxes and removing all taxes on the labour income. The results of the model are very intuitive.

Efficiency Gains in the UK from elimination of all taxes and transfers
(Measured as a percent of benchmark utility level of a representative household)

Equivalent Variation = 3.715
Compensating Variation = -3.582

Efficiency Gains from Switching to Labour income Taxes

Equivalent Variation = -6.9
Compensating Variation = 7.0

Efficiency Gains from Switching to Consumption Taxes
Equivalent Variation = 2.967
Compensating Variation = -2.882

The key results of this exercise in the general equilibrium impacts of tax reforms are the following:

(1)The efficiency gains from switching to only consumption taxes are about 80 percent of the gains of eliminating all the taxes. Optimal consumption tax rate given the revenue constraint set equal to 80 percent of the benchmark revenue level is 2.9 percent. This seems a very sensible result considering the fact that consumers ultimately bear the burden of all taxes. Similarly consumers make a choice whether to consume a certain product or not depending upon its price. If the prices are high because of taxes they can increase utility by not consuming the heavily taxed good and by taking more leisure instead of work.

(2) Labour income tax is highly distortionary in this model for various reasons. As before 47 percent tax rate of labour income is optimal to meet the required revenue target. It reduces the labour supply. Both output and consumption becomes smaller after such a tax is imposed. The efficiency losses from switching to this sort of taxes can be up to 6.9 percent of the original utility.

(3)The first result shows that the net deadweight loss of the current tax and transfer system is about 4 percent of GDP.

### 7.3.5 Conclusion

An analytical solution is provided to a simplest possible general equilibrium model, with households, firms and the government, to analyse how the price and real wage rate are determined in equilibrium and to assess the macroeconomic impacts on consumption and labour income taxes in an economy. In a numerical example, where a given revenue need to be raised for transfers, the welfare cost and macroeconomic impacts are higher when both consumption and income taxes are applied together than when only one of these two taxes are in use because of the compounding effects of taxes. When revenues are returned to households in the form of lump sum transfers the labour income taxes are found more distortionary than the consumption taxes.

The model is applied to assess three policy reform scenarios in the UK and found that switching to only consumption taxes would improve the efficiency in the economy which are about 80 percent of the welfare gains from the elimination of all tax and transfers and far better than switching completely to the labour income taxes. Since the ultimate burden of taxes lies on consumers, consumption taxes remove the inefficiency caused by the tax system more than any other taxes[1].

[1]Acknowledgement: I appreciate a lecture from Professor Stephen Parente then at Northeastern and some comments from Professor Richard Green in the earlier draft of this paper.

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# 8 Qualitative analysis

It is a research method that utilises, categorises and conceptualises meanings through words expressed in interviews, field visits, memos/diaries, photos and research observation. Interviews, observation, memos/ diaries, field notes, photos/videos, interim Summaries provide data that helps to raise questions in an issue, create testable hypotheses and get answers to an unknown phenomenon. This is grounded theory. Researcher has direct personal contact with the objects of research and can be subjective in interpreting the available evidences and getting conclusions and recommendations.

Table 23: Differences between Qualitative and Quantitative Methods

| Qualitative | Quantitative |
|---|---|
| Words | Numbers |
| Participants view point | Researchers view |
| Formulation of theory | Testing Theory (F, T tests) |
| Process | Static |
| Proximity of researcher | Distant researcher |
| Deep, psychological | Reliable data |
| Micro | Macro |
| Micro | Behavior |
| Meaning/subjective | Objective evidence |
| unstructured | structured |

**Steps for qualitative analysis**

1. Interactive data:
    - interview and field visits
    - (tapes, videos, photos scans)
2. Data processing (coding)
3. Introducing units (open)
4. Creating categories (Axial)
5. Relationship between categories (Selective)
6. Generating models
7. Testing models
8. Narrative analysis
9. Writing reports/articles

**Computer Aided Qualitative Data Analysis Software (CAQDAS)**

1. Systematic management of data in internal files, external files and memos
2. Queries – data bits and frequency
3. Coding (Nodes, Sets, Links, Models)
4. Sets (cases)
5. Nodes (free, Tree, cases)
6. Classification –
7. Relationship and Attributes
8. Dynamic and static models

**Secondary Data in Qualitative Analysis** Secondary analysis offers rich opportunities not least because the tendency for qualitative researchers to generate large and unwieldy sets of data means that much of the material remains under-explored.

Qualidata, an archival resource centre
web site: www.essex.ac.uk/qualidata
its online catalogue - Qualicat—can be searched at the following address:www.essex.ac.uk/qualidata/data/catinput.htı

**Examples of Large Scale ESRC Studies with Qualitative Data** Adult Learner Education Project (Lancaster-Liverpool and Blackburn) Nvivo file: Adultlearner.nvp (Sept 2002 –March 2006); 142 Semistructured interviews; Blackburn-Lancaster-Liverpool;

Data from Barton, D. et al. (2008)
Consumer behaviour project in Portsmouth (2500 interviews)
Nvivo file: Retails.nvp
Data from Clarke, I., Jackson, P. and Hallsworth, A. (2004)
Housing and Bank and Mortgage –Durham University (Nvivo file: Housing_Bank.nvp (December 4-Feb 2007); 119 case studies
Smith, S.J., Searle, B. and Cook, N., (2008)
Volunteer to Disneyland Project (volunteer.nvp) from the Bryman and Bell (2008) ; Snowball Interviews

**A Simple Example of Qualitative Survey** Survey on use of Mobile Phone

Q1. Why do you like a mobile phone?
Q2. Which is your favourite brand?
Q3. When do you need this?
Q4. How can this help you in your daily work?
Q5. What features do you like to be added in it?
Q6. What is the maximum monthly amount you like to pay for a mobile phone?

### 8.0.6 Non-parametric techniques

Statistical and econometric tests discussed above are derived ultimately from the Normal distribution that is based on true parameters of the population distribution such as mean, variance and higher moments. Nonparametric tests apply to much wider range of shapes of population distribution. Non-parametric tests use simple statistical concepts such as signs of measurements, order of relationships or category of frequencies. These are distribution free tests that include Wilcoxon (1945) test, Mann and Whitney (1947) , rank tests, Kruskal-Wallis Tests.

There are many non-parametric and non-linear techniques available for research.

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## 8.1 Triangulation of research methods

The major objective of research to find out the truth about the problems that is bothering, individual households, corporations, communities, policy makers in the local and national governments or the international community as a whole. Some questions are quantitative by nature such as the distribution of income, employment level by sectors, prices and costs of commodities, demand and supply of various goods and services in the economy, international trade, growth rates of output employment, capital stock, investment, rate of returns on financial assets, primary secondary or the university level education. Others are qualitative. These range about the welfare and just society, philosophical issues relating to the behavioral and psychological analysis of decision making process by individuals and firms or the policy makers that often involve abstract reasoning. If there is a solid proof of certain aspects of life found by quantitative method that should be validated also by qualitative method. This kind of cross check between research method is knows as triangulation. It is good research practice to triangulate findings. Analysis and conclusions established by quantitative methods can be cross checked by qualitative methods or otherwise. Real purpose of research is finding something new that helps in making good decisions.

# 9 Writing up the research report

Writing is very important part of research. Arguments should flow smoothly between sections and chapters. Logical sequence should be maintained. Motivations and justifications of study, aims and objectives of research should be clearly defined in the first chapter. It should followed by a critical review of the literature in the next chapter. Methodology of research should be guided both by the research objectives and by the review of the literature. Analytical structure should contain both quantitative and qualitative aspects to some expects. Theoretical and empirical modelling structure should match the topic and aims and objectives of research .Good progress in writing requires clarity in the framework and formulation of hypotheses regarding the cause and effect relationship among variables relating to the study. Some idea of results expected from the research should be considered before doing theoretical or empirical analysis or before plunging to collect the primary or secondary data. Researcher need to pay close attention to following nine points:

1. A good focus in the topic through out the research process is vital for a successful dissertation. Narrow down a topic to make it specific. The depth of analysis counts more on the quality of dissertation than its breadth.

2. Study should show that enough efforts have been made in the critical review of the relevant literature by consulting academic databases such as the Econlit and JSTOR. It may be impossible to review all works but must review the top most articles in the topic and the be up to date. Assess, summarize, criticise, examine, question, challenge and digest the theoretical and empirical arguments found in the literature and show originality by comparing and contrasting various findings. Project your own analyses and views regarding the issue based on the literature review concisely.

3. Clear presentation of the relevant theory applicable to the issue at hand is very important. A theory establishes links among a number of variables and helps to predict with some degree of confidence how a certain change in one variable can affect the value of another variable(s). Diagrams and/or equations for quantitative analysis or flow charts for qualitative analysis can be starting point of describing the underlying model. More challenging dissertations require more sophisticated model but simple and thorough model is better.

4. Explain each diagram and equation clearly and discuss how it proves your points. Use relevant cross section, time series or survey data to be used for the study and plot their levels, shares, ratios, proportions, find correlations, regressions or simulations as appropriate to see the patterns and to convince yourself whether they support the theoretical reasoning or hypotheses. Extend the number of years, or enlarge the size of the sample if necessary. Express ideas in short rather than long sentences and avoid jargons as far as practicable. Write one idea in one paragraph. Make sure that arguments flow smoothly among paragraphs. Divide the study in sections in natural order of organisation and give some layout or plan of the study in the beginning so that any reader of the thesis can follow the whole presentation without any difficulty.

5. Topics and issues should be introduced smoothly and gently. Write only meaningful sentences and paragraphs. It pays to rely on simple methodology that is thoroughly understood. More complicated specification of simulation or econometrics methods or surveys follow from simple logic. Which method is suitable and appropriate for a study depends partly on the objectives and questions of research as well as on the expertise, beliefs, interest and skills of the researcher. Whatever is presented should be professionally convincing to readers supported by evidences and original computation, calculations or abstractions.

6. It is very important to acknowledge ideas borrowed from the literature and referencing to others works should follow a standard format. For a journal article and a book site as Ramsey (1928) or Keynes(1936) in the main text of dissertation and provide a complete list references for each source used as following:

    - Ramsey, F.P. (1928) A Mathematical Theory of Saving, Economic Journal, 38, December, 543-559.
    - Keynes J. M. (1936) The General Theory of Employment, Income and Interest Rate, Cambridge University Press, London.

7. Check spelling and grammar and rules of punctuation and quotations. Follow rules of quotations and punctuations to avoid any allegation of plagiarism.

8. Actual presentation of research results is very important. Findings should be summarised in tables and discussed in summary style. These should be based on the actual research carried out for this study.

9. Edit the paper several times before submission. Do not hesitate to cut down irrelevant sections or put details in the appendix. Be consistent throughout the manuscript. Use summary rather than in extensive style to economise on space available.

# 10 Some Ideas for Research Proposals

# 11 Example 1: Dynamic Economic Models for Policy Analysis: EU and the US

This study aims to present a dynamic general equilibrium models for the US and the United States. We include large and small economies of EU including France, Germany, Spain, Italy and UK and countries such as Greece and Cyprus. These represent more 250 million people, more than 50 percent of EU's 500 million population. Then we model the US economy and the link between the EU and the US. First we build the dynamic CGE model of Euro area as in Bhattarai (2014a, b) and the US economy as in Bhattarai, Haughton and Tuerck (2015) in order to study the growth and redistribution impacts of changes in fiscal policies in these economies. Then we will construct a DSGE model of these economies to assess the business cycle impacts of policy spill overs between the EU and the US in the context of recent development in the global economy.

Several studies relating the single market project for European integration will be reviewed. For instance, Allen, Gasiorek, Smith, Flam, Sørensen (1998) had used inputs from an econometric model in a CGE model to assess the direct effect of reduction in trade barriers under the single market project. Their conclusion that the SMP had pro-competitive impacts and not only the nature but also the intensity of competition has increased on various industries in EU has sound theoretical basis. Baldwin, Francois, Portes, Rodrik, (1997) estimated costs and benefits of joining the EU and argued that risk premium of investment has reduced significantly after the implementation of the EU enlargement project. Their CGE model contained scale economies and Dixit-Stiglitz type monopolistic competition and model using a small open economy model that was calibrated to the GTAP database. Broer, Westerhout, Bovenberg (1994) analysed how reducing the burden of income taxes and pay as you go (PAYG) contributions from labour income can improve labour supply, generates more efficiency, incentives and Pareto improvement. Macroeconomic stability, higher economic growth, greater efficiency in allocation of resources and more equal distributions for peace and prosperity are building blocks of the European economic policy. From the Treaty of Rome to the adoption of the Lisbon Agenda 2000 the European Union has made significant improvements in integration among its 28 member countries and European economy is becoming the most powerful, dynamic and vibrant economy in the global economy.

Francois (1996) illustrated how higher rate of population growth in developing countries can erode real wages in developed countries in factor based trade models and two way trade based on product differentiation. Haaland (1992) finds 1992 SMP project to have positive impact through enhancement of capital accumulation. Harrison, Rutherford, Tarr (1997) find the benefit of the

Uruguay Round of trade negotiation between 96 to 171 billion in a GE model of 24 regions and 22 commodities. Haaland, Norman, Wergeland, Rutherford (1987) study comparative advantage under Ricardo- Hecksher-Ohlin-Jones framework in six region global economy model. Jensen and Rutherford (2002) explained how public debt reduction achieved through spending cuts hurts elderly poor though public goods and transfers provided by surplus generated by debt reduction would benefit future poor, thus intergenerational equity is likely to pose a threat to the fiscal consolidation that is less likely to occur. Keuschnigg, Kohler, Casella, Sapir (1996) in overlapping generation model evaluate access into the EU which have impacts on expected capital accumulation, saving and investment activities, trade integration and effects of adoption of common agricultural policies, though the net gains are around 1.24 percent of GDP. The accession treaty favours old and future generations at the cost of current generation. Nordhaus , Yang (1996) presented a regional integration model of climate change and the economy and consider pure market solutions, efficient cooperative outcome and non-cooperation equilibrium. Emission is controlled more under the cooperation rather than in the non-cooperative solution though high income countries may be major losers from cooperation. Piazolo (2001) incorporates adjustment cost of investment in order to capture the non-steady state phenomenon in the benchmark in a standard CGE model. Saito (2004) illustrates how estimation of elasticities of substitution be different when estimated with bilateral rather than multilateral data. Wren-Lewis, Darby, Ireland, Ricchi (1996) compare econometric macroeconomic model to a simpler theoretical model and continue perturbations until properties match between econometric and theoretical models for analysis of fiscal policy under the COMPACT model.

Wright (1988) studies stochastic economy with labour contracts with overlapping generations and finite horizon employers to reconcile data with equilibrium theory. Krusell, Ohanian Ríos-Rull, and Violante (2000) show how skill biased technological changes is the main reason for the rising gap in wages of skilled and non-skilled workers. Despite so much work on the European economies there are very few applied dynamic general equilibrium models available in the literature appropriate for analysing growth prospects in multi-household multisectoral set up to build and update the dynamic general equilibrium models the UK, Germany, France, Spain, Italy, Poland and other EU economies for evaluation tax, trade, income redistribution and growth, environmental, energy, conflict and growth policy issues.

### 11.0.1 Objectives of Research

The main objective of this project is to create a research framework which a decision makers want in a real world situation exploiting general equilibrium models which are based firmly on strategic considerations supported by econometric analyses. It draws on our research experience particularly in the University of Hull. The main aim of the project is to bridge the gap and improve the boundary of knowledge in applied general equilibrium models to analyze a policy issue under investigation using them simultaneously to highlight on current economic problems and is based in the following review of each of these modeling paradigms. The proposed project aims for

1. Some preliminary work has already been undertaken to develop a multi-household, multi-period and multi-sectoral dynamic general equilibrium model based on structural realities of the UK economy that can be used to evaluate the impacts of various supply and demand side policies on growth and reallocation resources among various sectors and redistribution of income.

2. It aims to apply these models successfully to analyse important policy issues facing the UK economy including growth and investment, energy and environment, savings, retirement and pensions, human and physical capital, unemployment and inflation, business cycles, sectoral and regional variations in level of income, consumption and trade, international competitiveness of the UK economy with respect to its EU trading partners such as Germany, France Spain and Italy and others such as the US and other emerging economies.

3. It aims to initiate close collaboration will be sought with the related government agencies including the Inland Revenue, the Treasury, the Department of Work and Pensions and the DTI.

4. It aims to generate cutting edge papers publishable in standard refereed journals. This modelling project is in fact the continuation of the research activities undertaken by this author under the EPSRC funded Supergen project for modelling the general equilibrium impacts of energy sector policies on growth, environment and redistribution in the UK economy at the University of Hull.

5. This project will seek to cultivate professional relationships within the UK and in Europe and in the North America through presentations in seminars and conferences, joint modelling projects and publication in standard journals.

### 11.0.2 Application of the Model

The completion of this model will provide a very good modelling framework for analysing many important policy questions.

1. It will be on analysis of impacts of tax and trade policies in growth and employment, investment and capital formation and technical advancement in next 50 or 100 years.

2. It will be applied to analyse issues of pension and social security problems using multi-household dynamic general equilibrium modelling framework.

3. It may provide consistent framework for analysing the link between energy and environment and growth. The 123 general equilibrium model constructed here and for analysis of energy sector policy issues under the Supergen project can be further improved and employed to examine how emission could rise as the production and consumption structure moves towards more polluting products and falls when the environment conscious households, firms and governments adopt behaviors that lead to less polluting goods and services.

4. It will be used to assess the issues of arrangement of trade and foreign direct investment with the EU and the US the major trading partners of the UK.

5. Finally it will extend the boundaries of knowledge in applied economic modelling by integrating the applied general equilibrium modelling with econometric and game theoretic models.

### 11.0.3 Dissemination of research output

After the completion of this research project it is expected to generate cutting edge journal articles, reports and book and letters. The major target journals for research output will include Economic

Journal, Applied Economics, Applied Financial Economics, Econometrica, Journal of Public Finance and Applied Econometrics. It will generate a number of research visits among the academic department and policy making institution in the UK and abroad for seminars and workshops. Research output will be presented in international conferences organised by Econometric Society, Royal Economics Society, European Economic Association, American Economic Association and similar other organisation. Collaboration between policy making institutions and academic departments, modelling schools and the Business School and the Centre for Economic Policy will be sought at regular intervals.

# 12 Example 2: Economic growth in India, China and SAARC countries

The process of convergence and divergence has been going on in the global economy in the last three hundred years after the scientific discoveries and technical innovations that have fundamentally changed the nature of production, exchange and consumption. Industrialised came to current stage going through stages of development from 18th to the last quarter of 20th century. This process has further intensified in the last six decades. Every country in the world wants to achieve a higher rate of growth of GDP per capita. While the countries in the West were successful in achieving higher growth till 1980s the growth pole has now gradually shifted towards the countries in developing Asia including India in the South Asia. Stylized facts of growth and economic development are presented here based on the data sets from the World Economic Outlook of the IMF and World Bank Development Indicators (WBDI).

The average growth rate in developing Asia has been 7 to 8 percent in the last 30 years, twice the global average and three times or more of that in the EU economies (Table 1). After decades of sluggishness, growth rates in South Asian countries have been higher than those in other regions of the world; particularly very impressive in India and China (Table 2). Bosworth and Collins (2008) provide growth accounting at aggregate and sectoral levels of the extraordinarily growth occurring in China and India, residence of over one third of the global population a lot more than less than 20 percent population residing in advanced countries.

Table 1: Average annual growth rate of GDP in the global economy (%)

| | ASEAN-5 | ADV Econ | CIS | CE Europe | DevAsia | EmDevEcon. | EuroA | EU | Majadv (G7) | MENA | MENAP | OthAdv | SSA | WestHm | World |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1980-89 | 5.30 | 3.12 | | 2.11 | 6.79 | 3.51 | | 2.15 | 3.03 | 1.47 | 1.99 | 4.73 | 2.60 | 2.12 | 3.24 |
| 1990-99 | 5.03 | 2.78 | -4.26 | 1.70 | 7.36 | 3.67 | 1.97 | 2.16 | 2.55 | 4.35 | 4.37 | 4.33 | 2.23 | 2.97 | 3.09 |
| 2000-09 | 4.87 | 1.78 | 5.98 | 3.90 | 8.31 | 6.15 | 1.35 | 1.75 | 1.45 | 5.42 | 5.34 | 3.37 | 5.53 | 3.18 | 3.62 |
| 2010-14 | 5.61 | 1.88 | 3.72 | 3.30 | 7.37 | 5.66 | 0.68 | 0.93 | 1.87 | 3.99 | 3.94 | 3.28 | 5.39 | 3.86 | 3.75 |

Table 2: Average annual growth rate of GDP in SAARC countries (%)

| | Afghanistan | Bangladesh | Bhutan | China | India | Maldives | Nepal | Pakistan | Sri Lanka |
|---|---|---|---|---|---|---|---|---|---|
| 1980-89 | | 3.28 | 9.37 | 9.76 | 5.54 | 10.52 | 4.10 | 6.59 | 4.21 |
| 1990-99 | | 4.80 | 5.33 | 10.00 | 5.63 | 6.61 | 4.87 | 4.50 | 5.61 |
| 2000-09 | 9.23 | 5.82 | 8.10 | 10.29 | 7.00 | 7.10 | 4.14 | 4.69 | 4.64 |
| 2010-14 | 6.72 | 6.15 | 8.66 | 8.46 | 5.81 | 4.33 | 4.25 | 3.34 | 7.13 |

Table 3: GDP per capita, current prices ($)

| | Afghanistan | Bangladesh | Bhutan | China | India | Maldives | Nepal | Pakistan | Sri Lanka |
|---|---|---|---|---|---|---|---|---|---|
| 1980 | | 236 | 321 | 307 | 277 | 413 | 138 | 374 | 301 |
| 1990 | | 284 | 544 | 341 | 386 | 1092 | 215 | 483 | 509 |
| 2000 | | 355 | 802 | 946 | 461 | 2967 | 247 | 581 | 917 |
| 2010 | 641 | 703 | 2063 | 4423 | 1432 | 6668 | 596 | 1034 | 2429 |
| 2014 | 641 | 1006 | 3042 | 7138 | 1389 | 7501 | 703 | 1234 | 3360 |

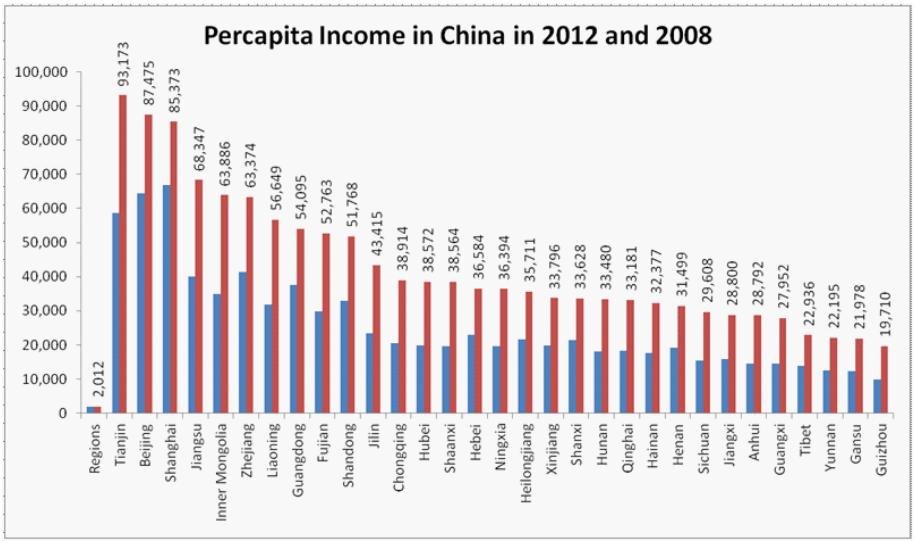
Size of the SAARC region has increased to around 7 percent of global GDP in PPP which more has more than doubled since 1980. However this growth in global share pales when compared to China raised its global share to 16.5 in 2014 percent compared to 6 percent of India. Srinivasan (2005) reports on TFP growth rates underlying these trends.

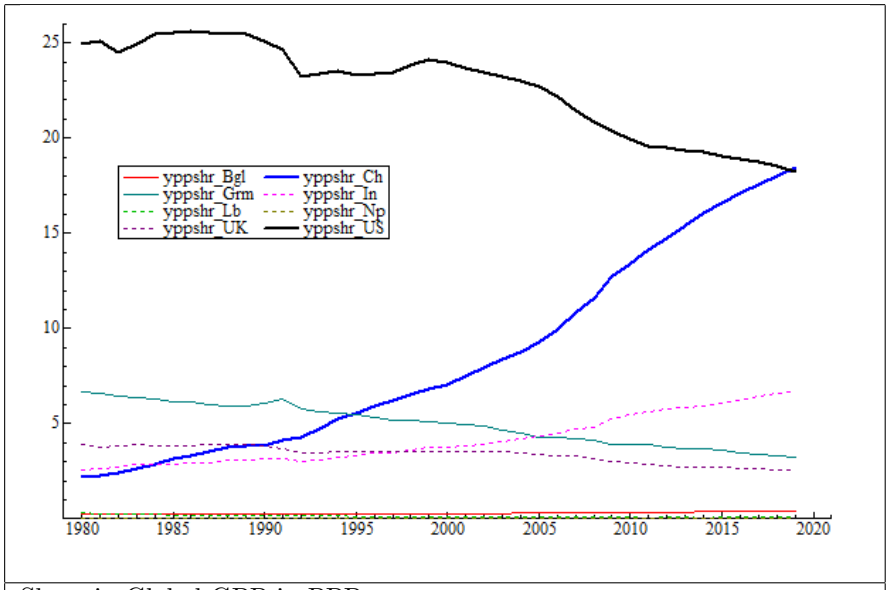
Table 4: Share of India and SAARC countries in the global GDP in PPP

| | Afghanistan | Bangladesh | Bhutan | China | India | Maldives | Nepal | Pakistan | Sri Lanka |
|---|---|---|---|---|---|---|---|---|---|
| 1980 | .. | 0.24 | 0.00 | 2.24 | 2.68 | 0.00 | 0.04 | 0.47 | 0.10 |
| 1990 | .. | 0.26 | 0.00 | 4.11 | 3.18 | 0.00 | 0.04 | 0.61 | 0.11 |
| 2000 | .. | 0.28 | 0.00 | 7.46 | 3.83 | 0.00 | 0.05 | 0.62 | 0.12 |
| 2010 | 0.04 | 0.35 | 0.01 | 14.10 | 5.66 | 0.00 | 0.05 | 0.65 | 0.15 |
| 2014 | 0.04 | 0.39 | 0.01 | 16.46 | 5.94 | 0.00 | 0.05 | 0.65 | 0.16 |

Economists generally agree on the factors that lead to economic growth as above based on experienced of Western Europe, North America, Japan and other advanced economies. Policies that raise the rate of accumulation of physical and human capital and advancement in the production technology leads to higher economic growth (Madison (1995)). Classical, neoclassical and endogenous growth models have been constructed to show the precise relationship between these factors and economic growth. Early versions of South Asian growth models used by the Planning Commission of these nations were based on basic Harrod-Domar set up where given the capital output ratio increasing growth required just increasing the rate of national saving. Then there were various sectoral decomposition exercises aimed to fit the aggregate target. Big gaps remained between targets and accomplishments. Levels of per capita income were similar across all SAARC countries till 1980 but these started to differ substantially following the economic reforms and liberalisations that started in India in late 1980s (after the success of similar trend in China). Kotwal, Ramaswami and Wadhwa (2011) explain how the recent growth in India was spurred by exports of high tech services rather than manufacturing products as in China. Now the National Initiative for Transforming India (NITI) has replaced the Planning Commission of India from January 1, 2015. Similar reforms are taking place in China. It has become important to think how these new institutions will fit in the recents trends of growth and distribution and how they will be able to alleviate poverty from Asia in coming years.

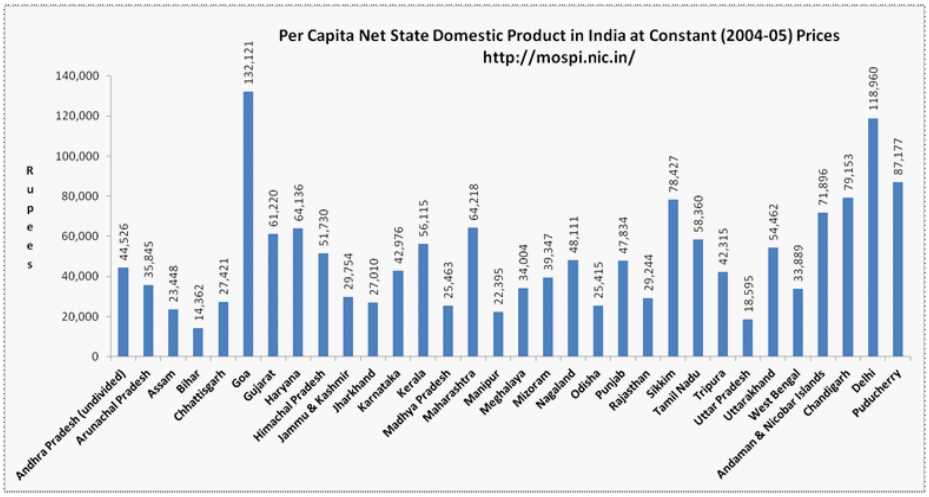
Percapita income in China in 2012 and 2008




Share in Global GPP in PPP

Systematic thinking about the process of economic growth in the South Asian economies started with the implementations of periodic plans in 1950s (Srinivasan (1964), Myrdal (1982), Sen (1983) ). Improvement in the national accounting and input output analysis in 1950s made it possible to focus on analysis of economic growth rates (Krishnamurty (1966), Swamy (1973)). These plans contained discussions about the strategies and programmes of the government for the development of agriculture, construction of road and telecommunication networks, industrialisation, literacy and numeracy, further education and health in process of human capital, adoption of the traditional technology of production, alleviation of poverty and many other issues. While government inter-




vention in the economy increased under the ISI strategy, these planning exercises also contained programmes for decentralisation (Bardhan (2002)). Whether to follow the labour intensive technology under the Ghandhian or Nehruvian theory of growth or capital intensive technology using the capitalist market economy were debated academically by (Srinivasan (1962), Sen (1968), Mahalanobis (1958)). There were debates on whether a nation should push for big heavy industries first letting the trickle down effects to take care of poverty of masses were strategic issues. Social and economic institutions are still not appropriate in South Asia for growth when compared with that in China which created these between 1950s to 1980s, particularly during period of cultural revolutions (Basu (2007)). Challenges remained not only in transforming surplus labour from the rural agricultural sectors to industrial sectors in the dualistic economic set up but also in disciplining them for the hard work in the industrial sectors (Lewis (1952), Myrdal (1972) Basu (2009) ). Macro, multisectoral and general equilibrium models were constructed for analysis (Parikh and Panda (1995), Fan (2002)). Governments actively intervened in the economy developing various state owned enterprises and increasing the role of state in every aspects of the economy. This resulted in massive build up of bureaucracy, red tapism and corruption in public life. This brought distortions in the efficient allocations of resources. Plans and programmes remained in the self of the planning commissions and could not get implemented resulting in dismal economic growth rates that barely averaged around 3 percent through out 1950 to 1980s (Ahluwalia (2002)). Growth could not occur in South Asia at a desirable space while other countries including the South Korea, Taiwan, Hong Kong and Singapore transformed themselves from developing to advanced economies during this period adopting good set of export oriented public private partnership approaches to the economic growth.

What were the golden keys for growth in China and India in recent years? Kotwal, Ramaswamy and Wadha (2011) attribute India's growth mainly to the export oriented strategies of high tech sectors. Xu (2011) attributes the magic of growth in China to its disciplined workers and a regionally decentralised authoritarian system in which "The central government has control over personnel, whereas sub-national governments run the bulk of the economy; and they initiate, negotiate, implement, divert, and resist reforms, policies, rules, and laws". There are very few studies focusing on the aggregate growth in the South Asia in other countries (Srinivasan (2005)).

Economic integration of the South Asian region must base on the strength of its members. India is stable, dynamic and economic power of the region. Bhutan and Maldives two tiny countries of the region are doing better economically by pursuing strategies appropriate to the vastly growing production sectors and middle classes in India. Bhutan is benefiting by proximity of India by developing a number of hydro power stations generating electricity to sell to India. Maldives is developing fast by tourism aiming at individuals in the growing middle income class in India. Bangladesh is achieving higher growth rate than before by exporting textiles but still caught in natural disasters and political problems. War torn Afghanistan and Pakistan could not emerge above the ethnic conflicts to focus on economic growth. Despite uprooting the age old monarchy and being able to restore the peace with Maoists it is an irony that Nepal is yet struggling to form a political consensus to draft a new constitution for the republic of Nepal. Given above potentials and absurdities a systematic study, particularly focusing on the role that India can play in development of the South Asia region has become an interesting topic of research, apparently very little is found on this in the existing literature.

The election of the pro-growth Modi government in India in May 2014 and its initiation of growth oriented policies have raised optimism for higher growth not only in India but also in all South Asian countries. India contributes to 80 percent of GDP and population of South Asia. It is marching on the path of rapid growth since pro-liberalisation policies were adopted in 1991. It has shown keen interest in harnessing the natural and human resources for economic development of the regions by taking international initiatives in establishing the BRICS Bank, concept of trans-Himalayan growth axis, road, rail and information networks to strengthen SAARC regional economic cooperation. It has provided vision and leadership for growth.

It has taken more than seven decades to come to this point. Many argue that these decades were lost in process of finding the right ideas, philosophies and techniques required for speedy economic growth in India. It is still the case in neighbouring countries such as Afghanistan, Pakistan, Nepal and Sri Lanka. India can contribute to create atmosphere for structural changes and development of economic and social institutions required for such growth through out the region. The actions for liberalisation and economic reforms now being discussed and expected to be implemented in near future can have far reaching and more transformative effects for the long run growth than those implemented in 1990s.

By maintaining an average of 8 percent growth, it is possible that India will catch up with the advanced countries in the West and East in terms of per capita income within a generation. Other SAARC member countries may be able to converge to India in per-capita income by taking appropriate actions to create stable institutions and socio-economic conditions required for growth. By the size of the economy and manpower-strength, India is the centre of the economic gravity with seven satellite countries surrounding it. Considering the growth success story of China since 1980s, which is the eastern neighbor of this region, it is very essential and beneficial to India to have an integrated approach for the development of these countries in South Asia. Modi's recent proposal for HIT-ways (highways, information technology and transmission ways) to fight against poverty in the region is a timely and visionary proposal for growth. Several strategic points for growth emerging from the analysis of facts in this paper are worth considering in this context. These are as follows:

1. Given that the 20 percent population resides in the South Asia this region should push for growth and increase its share of global GDP up to 20 percent from roughly 6.5 percent in 2014.

2. Such growth requires increasing the ratio of saving and investment about 10 percent above the current averages around 35 percent.

3. Process of structural transformation should continue so that output and employment increases substantially in industrial and services sectors and till both the output and employment in the agriculture sector are less than 5 percent from around 17 and 50 percent in recent years though both growing in absolute terms .

4. Such transformation will occur as this region moves towards urbanisation so than about 90 percent of the population starts living in urban area with facilities. Building mega cities like this will create not only employment but also income. It also will gradually free up rural lands for more scientific cultivations and other meaningful economic uses.

5. On manpower issues it is important to reduce the student teacher ratio from 40 to close to 16 to raise the quality of education and cognitive skill among children. This is essential for human capital required for science and technology.

6. Revenue and spending should balance at least in the medium term and debt to GDP ratio should not increase over 50 percent of GDP.

7. Trade ratio should increase to around 100 percent from the 50 percent at this time. Free trade regimes can enhance both the supply and demand sides of the economy.

8. Liquidity of the financial system need at least to treble to have a smooth flow of credits required for new and existing enterprises.Free convertibility of currency is essential to protect this region from international shocks.

9. Gini coefficient should not be above 35 percent for social integrity and cohesion.

10. A high 8 percent growth strategy is consistent with all above and requires firm commitment for economic development, efficient and strong public administration.

It has become very important to implement the social choice theory of Sen (1999) to enhance the welfare of the people in the South Asia under the process of rapid globalization. The objective of this paper is to identify models that are essential for the analysis of growth trajectories that may fit well to emerging stylized facts of the South Asian economies based on time series data and relevant literature[2].

## 12.1 Models of Growth and Development of India and South Asian Countries

A model is a systematic representation of an economy in a set of variables related to each other by parameters expressing the behavior of producers, consumers and policy makers. Economist use partial and general equilibrium models to study growth and development. This research project aims to devel models for systematic representations of above economies.

Firms produce output employing capital and labour inputs given the production technology to maximise profit. Some of them operate under more competitive markets and others in more monopolistic structure. These private firms are key drivers of economic growth as there are significant

[2]Data series used in this paper are obtained from the World Economic Outlook and International Financial Statistics of the IMF and the World Bank Data Indicators accessed through the data archive in the UK (UKdata.stat).

spill over effects within and between industries as firms engage in the research and development activities. Demand for their products by households change with the level of their income. Analysis of supply and demand by firms and the industry, pricing, costing and output decisions in response to the policies over time are usually analysed using partial equilibrium models. Elasticities of demand and supply or substitutions among inputs provide information in the structural features of these markets. Multiple regression, cross section and panel data analysis, AR, MA, ARIMA, ARCH GARCH models are applied to estimate parameters of functions derived from the optimisation of objectives of firms and households subject to budget or technology constraints. These partial equilibrium models have been applied to study the volatility of prices and trade in the financial markets and their consequences in the welfare of households (Fama (2014), Shiller (2014)).

General equilibrium solutions are obtained when all markets clear to a set of equilibrium relative prices consistent to Pareto optimal allocations across all these markets (Balasko and Geanakoplos (2012)).These models rest on the detailed information obtained from the input-output tables, tax-transfer system, social accounting matrices and national accounts of these economies. Then there are static and dynamic strategic models to analyse interdependence of activities of economic agents.

There is no single economic model that is perfect and fit for analysis of all important issues relating to growth and development. Each type of model has its strength and limitations. Since the overall objective is having a comprehensive understanding of underlying factors that influence on growth and development it is essential to consider each of these models and appreciate how it can contribute to our understanding of the economy. We illustrate this by applying a panel data model of growth, dynamic CGE model with financial deepening, macroeconometric model for macroeconomic forecastging and a policy coordination model to analyse gains from cooperation to enhance growth and development in India and SAARC countries in this section.

## 12.2 Dynamic Panel Data Model of Economic Growth

Growth models show how the output per capita increases over time with accumulation of physical and human capital and improvement in technology (Solow (1956), Lucas (1988), Romer (1990)). However the growth rates differ significantly by countries and the degree of convergence in per capita income varies substantially across nations. Frustrated from the dismal growth performance from 1950-1980s Malenbaum (1982) even stated pessimistically that "decades of slow growth lie ahead before either nation emerges as a modern industrial state of developed-nation status". Fortunately there occurred a structural break in the growth process around mid 1980s in India motivating Rodrik and Subramanian (2005) to assess policy and structural factors that caused a surge from "Hindu growth" to productivity surge. These surges occurred because of the reforms of the labour market giving freedom in hiring and firing of workers to firms, end of reservation in small scale industries, reforms of the banking sector, simplification of FDI rules, improvement in infrastructure and reduction of debt. These policy factors accelerated growth in India starting in early 1990s (Kaur (2007)). Agrawal (2010) empirically establishes causality between savings and economic growth in India. Bosworth and Collins (2008) provided growth accounting at aggregate and sectoral levels of the extraordinarily growth occurring in China and India. From the panel data analysis and endogenous growth models Basu and Bhattarai (2012a) found that cognitive skill and openness to be factors of higher economic growth. Shocks to the technology sectors caused more macroeconomic fluctuations than the total productivity shocks in the short run in their models. Education is the key for growth but it is the joint responsibility of public and private sectors to educate children. Public bias to education does not produce desired results (Basu and Bhattarai (2012b)). South

Asia forms the part of global economy in both of these endogenous growth models. We estimate coefficient the dynamic panel data model of growth for the South Asian economies report results in Table 14. This shows in general trade ratio and investment ratios contribute significantly and positively on the growth rates of per capita income but the higher population growth rates reduced output growth rates significantly. However there are country and time specific factors at play as growth rate vary significantly across countries and time years.

### 12.2.1 GMM 2-step Estimation of Growth in South Asia

Consider a dynamic panel data model of the form where growth rate of output of country $i$ at time $t$, $y_{i,t}$ is explained by its lagged values and a set of exogenous explanatory variables $x_{i,t}$. Here $\alpha_i$ is individual specific effects and $\lambda_t$ represents the time specific effects.

$$y_{i,t} = \gamma y_{i,t-1} + \alpha_i + \beta_i x_{i,t} + \lambda_t + + e_{i,t} \qquad \gamma < 1 \tag{556}$$

A generalised method of moments (GMM) as proposed by Hansen (1982) for a panel data model generates the unbiased estimate of $\gamma$ and $\alpha_i$ solving endogeneity and bias in estimation due to the presence of correlation between the lagged values of dependent variables $y_{i,t-1}$ and errors terms $e_{i,t}$. Right instrument for lagged $y_{i,t-1}$ say by $y_{i,t-2}$ solves this inconsistency and generates unbiased estimator (ignoring $x_{i,t}$ and $\lambda_t$):

$$\widehat{\gamma}_{IV} = \frac{\sum_{t}^{T}\sum_{i}^{N} y_{i,t-2}\left(y_{i,t-1} - \overline{y}_{i,t-2}\right)}{\sum_{t}^{T}\sum_{i}^{N} y_{i,t-2}\left(y_{i,t-1} - y_{i,t-2}\right)} \tag{557}$$

where $y_{i,t-2}$ is used as instrument of $(y_{i,t-1} - y_{i,t-2})$.
GMM method includes the most efficient instrument, $Z_i$:

$$\begin{aligned} \gamma_{GMM} \quad = \quad & \left(\left(\sum_{i=1}^{N} \Delta y_{i,t} Z_i\right) W_N \left(\sum_{i=1}^{N} Z_i^{'} \Delta y_{i,t}\right)\right)^{-1} \times \\ & \left(\left(\sum_{i=1}^{N} \Delta y_{i,t} Z_i\right) W_N \left(\sum_{i=1}^{N} Z_i^{'} \Delta y_{i,t}\right)\right) \end{aligned} \tag{558}$$

Arrelano and Bond (1995), Wijndmeir (2000), Blundell and Smith (1989) and Verbeek (2004), Wooldridge (2002) among others have more extensive analysis of the GMM estimation. The essence of the GMM estimation remains in finding a weighting matrix that can guarantee the most efficient estimator. This should be inversely proportional to transformed covariance matrix.

$$W_N^{opt} = \left(\left(\frac{1}{N}\right) \sum_{i=1}^{N} Z_i^{'} \Delta e_{i,t} \Delta e_{i,t}^{'} Z_i\right)^{-1} \tag{559}$$

The GMM estimator with instrument (levels, first differences, orthogonal deviations, deviations from individual means, combination of first differences and levels) used in PcGive is:

$$\widehat{\delta} = \left( \left( \sum_{i=1}^{N} W_i^* Z_i \right) A_N \left( \sum_{i=1}^{N} Z_i^{'} W_i \right) \right)^{-1} \left( \left( \sum_{i=1}^{N} W_i^* Z_i \right) A_N \left( \sum_{i=1}^{N} Z_i^{'} y_i^* \right) \right) \tag{560}$$

where $A_N = \left( \sum_{i=1}^{N} Z_i^{'} H_i Z_i \right)^{-1}$ is the individual specific weighting matrix.

Table 14: Panel estimates on the growth rate of per capita income in India and South Asia

| | 1-Step Estimation | | 2-step estimation | |
|---|---|---|---|---|
| Determinants | Coefficient | t-prob | Coefficient | t-prob |
| Trade ratio | 0.0025 | 0.0500 | 0.0038 | 0.0200 |
| Investment ratio | 0.0086 | 0.0080 | 0.0089 | 0.0040 |
| GDP growth rate | 0.9749 | 0.0000 | 0.9773 | 0.0000 |
| Population growth rate | -2.9055 | 0.0000 | –0.1275 | 0.0000 |
| Constant | -0.3575 | 0.0000 | 0.3424 | 0.0000 |
| T2005 | -0.1797 | 0.0590 | -0.1390 | 0.0790 |
| T2006 | -0.0435 | 0.4000 | 0.10750 | 0.4090 |
| T2007 | -0.0516 | 0.0320 | -0.0299 | 0.3790 |
| T2008 | 0.0208 | 0.0120 | 0.0439 | 0.0000 |
| T2009 | 0.1764 | 0.0060 | 0.1001 | 0.2210 |
| T2010 | 0.1353 | 0.0020 | -0.1493 | 0.0200 |
| T2011 | 0.1301 | 0.0000 | -0.1664 | 0.0000 |
| T2012 | 0.2791 | 0.0000 | 0.2791 | 0.0000 |
| Afghanistan | 0.2191 | 0.0000 | 0.0454 | 0.0000 |
| Bhutan | 0.0694 | 0.0000 | 0.0237 | 0.0020 |
| Bangladesh | 0.2200 | 0.0000 | 0.0554 | 0.0000 |
| India | 0.3371 | 0.0000 | 0.0350 | 0.0000 |
| Maldives | 0.1699 | 0.0000 | 0.0605 | 0.0000 |
| Nepal | 0.2823 | 0.0000 | 0.0135 | 0.0000 |
| Pakistan | 0.3466 | 0.0000 | 0.0742 | 0.0000 |
| Sri Lanka | base | base | base | base |
| | N =8; T=9 | $R^2$=0.99 | N =8; T=9 | $R^2$=0.99 |
| Data source: WBDI, IFS of IMF accessed from DataArchive UK | | | | |

Doornik and Hendry (2001, chap. 7-10) provide a procedure on how to estimate coefficients using fixed effect, random effect and the GMM methods including a lagged terms of dependent variable among explanatory variables for a dynamic panel data model: $y_{i,t} = \sum_{i=1}^{p} a_k y_{i,t-s} + \beta^t (L) x_{i,t} + \lambda_t + \alpha_i + e_{i,t}$ or in short $y_{i,t} = W_i \delta + \iota_i a_i + e_i$. It will be relevant to study process of convergence among states in India and SAARC countries using this type of growth model in coming years (see Brandt, Ma, Rawski (2014) for China).

## 12.3 Dynamic Computable General Equilibrium Model

One sector growth models presented above are analytically tractable but practically they are not designed to answer questions relating to sectoral structure of production, issue of structural transformation and distribution of income as an outcome of the general equilibrium process in the

economy. This requires a full dynamic computable general equilibrium (DCGE) model for a decentralised economy. DCGE models contain the relative price system and intertemporal choices of firms and households as key factors determining the growth of various sectors of the economy and distribution of income among households while studying the long run cycles of model economies (Bhattarai (2010)). The main equations for a typical DCGE model are as follows:

1) Demand side: welfare of households $\left(U_0^h\right)$ given by consumption $\left(C_{i,t}^h\right)$ and leisure $\left(L_t^h\right)$ :

$$Max \quad U_0^h = \sum_{t=1}^{\infty} \beta_h^t U_t^h; \quad 0 < \beta_h^t < 1 \tag{561}$$

$$U_t^h = U\left(C_{i,t}^h, L_t^h; \sigma_c\right) \tag{562}$$

Subject to budget constraints:

$$\begin{aligned} I_0^h &= \left[\sum_{t=0}^{\infty} e^{-\rho t} \sum_{i=1}^{N} \left\{P_{i,t}\left(1+t_i\right) C_{i,t}^h\right\} + w_t^h\left(1-t_l\right) L_t^h\right] \\ &= \sum_{t=0}^{\infty} e^{-\rho t} I_t^h = \left[\sum_{t=0}^{\infty} w_t^h\left(1-t_l\right) \overline{L}_t^h + r_t\left(1-t_k\right) K_t^h\right] \end{aligned} \tag{563}$$

2) Supply: production, finance and accumulation:

$$Y_{i,t} = F_i\left[K_{i,t}\left(r_{i,t}, w_t^h, p_{i,t}\right),\ p,\ L_i\left(w_t^h,\ p_{i,t}\right),\ A_i, \sigma_c\right] \tag{564}$$

$$\sum_{t=0}^{T} P_{i,t} Y_{i,t} = \sum_{t=0}^{T}\left[r_t\left(1+t_k\right) K_{i,t} + \sum_{h=i}^{H} w_t^h\left(1+t_l\right) L_{i,t}^h\right] \tag{565}$$

Savings $(Y_t - C_t)$ adds to the accumulation of assets $(A_t)$ in the economy:

$$A_t\left(1+\widehat{r}_t\right) + Y_t - C_t = A_{t+1} \tag{566}$$

$$A_t r_t + Y_t - C_t - \left\{A_{t+1} - \left(1-\delta\right) A_t\right\} = 0 \tag{567}$$

In equilibrium there is equivalence between financial assets $(A_t)$ and physical capital $(K_t)$ ; replace $A_t$ by $K_t$:

$$Y_t - C_t - \left(K_{t+1} - \left(1-\delta\right) K_t\right) = 0; \quad \Longrightarrow\Longrightarrow Y_t = C_t + I_t \tag{568}$$

This the optimal financial deepening at the sectoral and aggregate levels:

$$F_t = \frac{K_t}{Y_t}; \quad F_{i,t} = \frac{K_{i,t}}{Y_{i,t}}; \quad F_t = \sum_{i=1}^{N} F_{i,t}; \quad K_t = \sum_{i=1}^{N} K_{i,t}; \quad Y_t = \sum_{i=1}^{N} Y_{i,t} \tag{569}$$

3) Intetemporal balance:

$$\sum_{t=0}^{T}\sum_{i=1}^{N} P_{i,t}\left(1+t_{ci}^h\right) C_{i,t}^h = \sum_{t=0}^{T}\left[r_t\left(1-t_k\right) K_t^h + R_t^h + w_t^h\left(1-t_l\right) LS_t^h\right] \tag{570}$$

$$\sum_{t=0}^{T} P_{i,t} Y_{i,t} = \sum_{t=0}^{T} \left[ r_t \left(1 - t_k\right) K_{i,t} + \sum_{h=i}^{H} w_t^h L_{i,t}^h \right] \tag{571}$$

$$\sum_{t=1}^{T} G_t \lesseqgtr \sum_{t=1}^{T} \left( RV_t + \sum_{h=1}^{H} R_t^h \right) \tag{572}$$

4) Trade and finance:

$$\sum_{t=0}^{T} \sum_{i=1}^{N} PE_{i,t} E_{i,t} = \sum_{t=0}^{T} \sum_{i=1}^{N} PM_{i,t} M_{i,t} \tag{573}$$

$$\sum_{i=1}^{N} PE_{i,t} E_{i,t} - \sum_{i=1}^{N} PM_{i,t} M_{i,t} = \pm\ FL_t \tag{574}$$

5) Public sector and financial deepening:

$$\sum_{t=0}^{\infty} e^{-\rho t} RV_t \lesseqgtr \sum_{t=0}^{\infty} e^{-\rho t} \left( G_t + R_t^h \right) \tag{575}$$

$$RV_t = \sum_{h=1}^{H} \sum_{i=1}^{N} P_{i,t} t_{ci}^h C_{i,t}^h + \sum_{i=1}^{N} \sum_{h=i}^{H} \left( w_t^h t_l L_{i,t}^h + r_t \left(1 + t_k\right) K_{i,t} \right)$$

The general equilibrium is achieved when the excess demand are zero in each market for each period representing balance between demand and supply in each market. Households and producers optimise given their budget constraints. Relative price adjustment mechanisms guarantee the most efficient outcome in these markets. The existence of the general equilibrium is guaranteed by fixed point theorems and solved using the dynamic routines in the GAMS/MPSGE software. Given the properties of demand and supply functions equilibrium is stable and unique and gives the evolution of the model economies from 2006 to 2101(see Bhattarai (2007) and Bhattarai (2011)).

This model has been applied to China, India to study optimal and actual capital deepening ratios (OFDR and AFDR) and the results are summarised in Table 15. These show that the optimal capital intensity in China at 0.81 is much lower than in India's 1.54. This implies India economy being more capital intensive than the Chinese economy in production technology. However the ratio of actual stocks of the financial assets to GDP is much higher in China at 1.88 compared to 0.78 in India. Thus China is over-financed with over financing ratio (OFR) at 2.3 and India is under-financed with the OFR at 0.49. This result implies speedy growth in India requires a rapid growth of its financial sector (see Dougas and Rajan (2008) and Kawai (2011)).

Table 15: Optimal and actual financial deepening ratios and Growth Rates for 2008-12

| Countries | OFDR | AFDR | OFR | GR 2008-12 |
|---|---|---|---|---|
| China | 0.81 | 1.88 | 2.3 | 9.30 |
| India | 1.54 | 0.78 | 0.49 | 6.50 |

Note: OFDR and AFDR are optimal and actual financial deepeing ratios; OFR over financing ratio

Based on Bhattarai (2014); More details available upon request.

Main focus of this DCGE model is to study the long run growth in output and employment across sectors given endogenous or exogenous changes in the rate of taxes and tariffs. Comparative static features of Parikh, Narayana, Panda and Kumar (1995) could be put in such dynamic frameworks to study the evolution of Indian economy in coming decades. GTAP and GTAPinGAMS models also could be applied for empirical investigation on equilibrium relations among all South Asian economies to test theories of Bhagwati and Srinivasan (2002), Panagaria (2006), Neary (1998) for assessing how these countries benefit from inter and intra regional trade. Various arrangements for creating free trade area (FTA) under the SAPTA or other bilateral agreements can be studied constructing small open economy or multicountry trade models. Opening economies for trade with specialisation based on comparative advantages are essential features of the growth competition. A free trade association (FTA) under the South Asia Free Trade Association (SAFTA) can open such opportunities of cross boarder production and trade. India can sell skill, technology and manufacturing goods to its neighbors; it can buy cheaper hydro electricity from Nepal and Bhutan and agricultural products from Pakistan. Gains from cooperative rather than discriminatory approach with respect to the rest of the world could be used for the development of the region. Given the development of the GTAP/Unido/STAN databases it is possible now to analyse the significance of bilateral and multilateral trade relations among these countries. As opening intra-regional FDI could increase productivities, it is essential to remove limited product coverage, existence of negative lists and restrictive rules of origin that are becoming obstacle in such settings (Taneja and Sawhney (2007)).

## 12.4 Macroeconomic simulation models of South Asia

With time series on major components of aggregate demand, price levels, interest rate and exchange rates presented above it is possible to construct a macroeconometric model to forecast macro variables of India and South Asian economies. Essentially these models are helpful in studying trends and forecasts in the short run specially useful for annual projection of macro quantities such as consumption, investment, imports or exports or public spending and prices in the private and public sectors given projections of the public finance or the BOP conditions of the economy. Each South Asian economy have some sorts of open economy IS-LM model underlying their policy decisions and assessing the macroeconomic fluctuations. These basically Keynesian demand driven models are popular as they are easier to compute and implement because of recent innovations in econometric techniques (Hendry and Doornik (1994), Bhattarai (2008) and Bhattarai and Mallick (2013)). We estimate simultaneous equations models of India, China and SAARC countries to study how inflation, current account balance and growth rates relate to fiscal and monetart policy variables represented by the size of the government (g_y) ad liquidity ratio (M2_y) and structural facture (a_g). Again results presented in tables 16 to 19 below show significance ($t\_prob$) and sign of coefficients ($\beta$) on them vary tremendously across these countries. This means markets and policies are very different among these countries.

Table 16: Macroeconomic model of inflation, current account and growth in India and Nepal

| | India | | | | | | Nepal | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Inflation | | CA balance | | Growth | | Inflation | | CA balance | | Growth | |
| | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ |
| g_y | 0.084 | 0.223 | -0.007 | 0.985 | 0.007 | 0.904 | -0.494 | 0.017 | 0.017 | 0.192 | -0.111 | 0.463 |
| a_g | 1.417 | 0.001 | -6.275 | 0.008 | -0.138 | 0.667 | 0.258 | 0.049 | -0.002 | 0.812 | -0.038 | 0.693 |
| M2_y | 0.455 | 0.002 | -3.337 | 0.000 | -0.002 | 0.985 | 0.251 | 0.030 | 0.003 | 0.672 | 0.032 | 0.714 |
| const | -0.53 | 0.005 | 321.9 | 0.003 | 9.626 | 0.518 | -10.44 | 0.229 | -0.164 | 0.780 | 5.377 | 0.441 |
| | $R^2$=0.66; N =36; F(9,65) = 5.99 [0.0000] ** | | | | | | $R^2$=0.59; N =36; F(9,65) = 3.29517 [0.0023] ** | | | | | |

Table 17: Macroeconomic model of inflation, current account and growth in Bangladesh and China

| | Bangladesh | | | | | | China | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Inflation | | CA balance | | Growth | | Inflation | | CA balance | | Growth | |
| | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ |
| g_y | 1.713 | 0.003 | -0.006 | 0.962 | -0.436 | 0.003 | -0.066 | 0.689 | 3.058 | 0.247 | 0.178 | 0.031 |
| a_g | 0.925 | 0.003 | -0.050 | 0.568 | -0.213 | 0.021 | -1.357 | 0.057 | -13.24 | 0.232 | -0.775 | 0.026 |
| M2_y | 0.133 | 0.189 | 0.013 | 0.595 | 0.018 | 0.489 | -0.210 | 0.075 | -0.603 | 0.739 | -0.127 | 0.027 |
| const | -44.46 | 0.004 | 0.655 | 0.858 | 15.66 | 0.000 | 54.49 | 0.036 | 339.31 | 0.349 | 35.93 | 0.005 |
| | $R^2$=0.87; N =36; F(9,65) = 9.93091 [0.0000] ** | | | | | | $R^2$=0.66; N =36;F(9,65) = 4.07898 [0.0003] ** | | | | | |

Table 18: Macroeconomic model of inflation, current account and growth in Pakistan and Sri Lanka

| | Pakistan | | | | | | Sri Lanka | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Inflation | | CA balance | | Growth | | Inflation | | CA balance | | Growth | |
| | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ |
| g_y | 0.187 | 0.055 | -0.129 | 0.145 | -0.180 | 0.000 | 0.023 | 0.812 | 0.007 | 0.696 | 0.035 | 0.409 |
| a_g | 0.321 | 0.368 | 0.064 | 0.844 | -0.131 | 0.416 | -0.074 | 0.772 | 0.058 | 0.003 | -0.131 | 0.288 |
| M2_y | -0.284 | 0.178 | 0.077 | 0.612 | 0.238 | 0.016 | -0.582 | 0.184 | 0.031 | 0.094 | -0.154 | 0.422 |
| const | 10.184 | 0.478 | -6.446 | 0.624 | 0.051 | 0.969 | 32.929 | 0.103 | -8.940 | 0.015 | 12.560 | 0.152 |
| | $R^2$=0.56; N =36; F(9,65) = 2.9261 [0.0057] ** | | | | | | $R^2$=0.42; N =36; F(9,65) = 1.86883 [0.0726] | | | | | |

Tabble 19: Macroeconomic model of inflation, current account and growth in Bhutan and Maldives

| | Bhutan | | | | | | Maldives | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Inflation | | CA balance | | Growth | | Inflation | | CA balance | | Growth | |
| | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ | $\beta$ | $t\_prob$ |
| g_y | -0.088 | 0.157 | -0.002 | 0.324 | 0.185 | 0.053 | 0.116 | 0.203 | -0.004 | 0.008 | -0.164 | 0.039 |
| a_g | -0.167 | 0.206 | 0.007 | 0.073 | -0.270 | 0.177 | -0.996 | 0.009 | 0.008 | 0.146 | 0.260 | 0.399 |
| M2_y | -0.157 | 0.020 | 0.001 | 0.452 | -0.118 | 0.233 | 0.047 | 0.713 | -0.009 | 0.000 | 0.009 | 0.932 |
| const | 22.34 | 0.001 | -0.265 | 0.161 | 12.81 | 0.171 | 6.073 | 0.169 | 0.318 | 0.000 | 10.385 | 0.009 |
| | $R^2$=0.55; N =33; F(9,65) = 2.89281 [0.0061] ** | | | | | | $R^2$=0.79; N =33; F(9,65) = 6.78843 [0.0000] ** | | | | | |

The business cycle analyses in DSGE models contrain micro-foundations, dynamics and rational expectations, stochastic shocks to preferences, technologies and policies along with the nominal and real rigidities than present in above models. Analysis of short or long run multipliers, variance decompositions and impulse responses to changes in policies and shocks on the deviations of model variables from the steady state are often the focus of such analysis. Computations have become easier for such models after development of Sim's BVAR algorithm in the MATLAB and dynare. However we skip this model here as the growth and redistribution analysis in the DCGE model presented above is better suited for analysis of structural features of the South Asian economies

than these DSGE models.

## 12.5 Strategic policy coordination models of South Asia

Interdependence among these economies and interactions could be studied using bargaining, signalling and mechanism designing concepts. Cooperative and non-cooperative games with complete and incomplete information among nations, households and firms could be used to conceptualize the issues and solutions to the problems of growth and development in these economies. There are three generations of literature in the policy coordination. First generation models include studies such as Kydland and Prescott (1977), Driffil (1988), Currie and Levine (1986) and Obstfeld and Rogoff (2000). These had found gains from coordination to be small. Cooper (1969) and Hamada (1976) and Kydland (1975) showed inferiority of the non-cooperative Nash equilibrium compared to a cooperative solution. Lucas (1976), and Kydland and Prescott (1977) used rational expectations and argued for the advantage of rule-based policies to create rational expectations equilibrium solution. Petit (1989) used differential games as did the studies of Obstfeld (1994), Sutherland (1996), Senay (1998), Martin and Rey (2000). Obstfeld (2001) and Rogoff (2002) provide an excellent review of some of the models used for policy coordination with Mundell-Fleming-Dornbush type models with little gains from coordination. Second generation models of policy coordination in Pappa (2004), Canzoneri, Cumby and Diba (2005), Clerc, Dellas and Loisel (2011), Juillard and Villemot (2011) and Goyal (2007) find pay off from monetary and fiscal policy coordination to be bigger. Supply and strategic modelling has much improved in recent literature on the policy coordination showing more gains from coordination as stated by Conzoneri et. al.(2005), Evans and Hnatkovska (2007), Douglas and Laxton in dynare. Aarle et.al. (2002) examine the coalition formation in EMU. Recent models such as Kempf and von Thadden (2013), Dedola et al. (2013) add asymmetric information and commitment where the welfare gains can be bigger as the number of countries increase in such deals. Given this literature let us consider three countries aiming for a policy coordination with the Nash utility frontier:

$$N_t = U_{1,t}U_{2,t}U_{3,t} \tag{576}$$

Each receive utility from consuming products produced in each country:

$$U_{i,t} = F\left(y_{1,t}, y_{2,t}, y_{3,t}\right) \tag{577}$$

Goods supply process is determined simultaneously as:

$$y_{1,t} = \alpha_{1,0} + \alpha_{1,2}y_{2,t} + \alpha_{1,3}y_{3,t} + \beta_{1,1}y_{1,t-1} + \beta_{1,2}y_{2,t-1} + \beta_{1,3}y_{3,t-1} + e_{1,t} \tag{578}$$

$$y_{2,t} = \alpha_{2,0} + \alpha_{2,1}y_{1,t} + \alpha_{2,3}y_{3,t} + \beta_{2,1}y_{1,t-1} + \beta_{2,2}y_{2,t-1} + \beta_{2,3}y_{3,t-1} + e_{2,t} \tag{579}$$

$$y_{3,t} = \alpha_{3,0} + \alpha_{3,1}y_{1,t} + \alpha_{3,2}y_{2,t} + \beta_{3,1}y_{1,t-1} + \beta_{3,2}y_{2,t-1} + \beta_{3,3}y_{3,t-1} + e_{3,t} \tag{580}$$

Coefficient of a VAR model estimated from the time series data provides information on interactions among model economies as:

$$\begin{pmatrix} 1 & -\alpha_{1,2} & -\alpha_{1,3} \\ -\alpha_{2,1} & 1 & -\alpha_{2,3} \\ -\alpha_{3,1} & -\alpha_{3,2} & 1 \end{pmatrix} \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix}$$
$$= \begin{pmatrix} \alpha_{1,0} \\ \alpha_{2,0} \\ \alpha_{3,0} \end{pmatrix} + \begin{pmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-2} \\ y_{3,t-3} \end{pmatrix} + \begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{pmatrix} \tag{581}$$

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix}$$
$$= \begin{pmatrix} 1 & -\alpha_{1,2} & -\alpha_{1,3} \\ -\alpha_{2,1} & 1 & -\alpha_{2,3} \\ -\alpha_{3,1} & -\alpha_{3,2} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \alpha_{1,0} \\ \alpha_{2,0} \\ \alpha_{3,0} \end{pmatrix} +$$
$$\begin{pmatrix} 1 & -\alpha_{1,2} & -\alpha_{1,3} \\ -\alpha_{2,1} & 1 & -\alpha_{2,3} \\ -\alpha_{3,1} & -\alpha_{3,2} & 1 \end{pmatrix}^{-1} \begin{pmatrix} \beta_{1,1} & \beta_{1,2} & \beta_{1,3} \\ \beta_{2,1} & \beta_{2,2} & \beta_{2,3} \\ \beta_{3,1} & \beta_{3,2} & \beta_{3,3} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-2} \\ y_{3,t-3} \end{pmatrix}$$
$$+ \begin{pmatrix} 1 & -\alpha_{1,2} & -\alpha_{1,3} \\ -\alpha_{2,1} & 1 & -\alpha_{2,3} \\ -\alpha_{3,1} & -\alpha_{3,2} & 1 \end{pmatrix}^{-1} \begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{pmatrix} \tag{582}$$

Paramters of VAR could be interpreted in the context of Nash Policy Game as:1) In common meetings or summits they decide policies given by $\alpha_{1,0}, \alpha_{2,0}, \alpha_{3,0}$ but each of them face idiocyncratice shocks $e_{1,t}, e_{2,t}, e_{3,t}$; 2) Then each country determine its action $y_{i,t}$ taking account of actions taken by others $y_{j,t}$ and such response patterns are given by parameters $\alpha_{1,2}, \alpha_{1,3}, \alpha_{2,1}, \alpha_{2,3}, \alpha_{3,1}, \alpha_{3,2}$ ,$\beta_{1,2}, \beta_{1,3}, \beta_{2,1}, \beta_{2,3}, \beta_{3,1}, \beta_{3,2}$ and shocks $e_{1,t}, e_{2,t}, e_{3,t}$; 3)Each would like to get more utility and this opens the bargain; 4) The optimal solution of this game should fulfill four properties of Nash bargaining game; 5) This must be symmetric, efficient, linear invariance and IIA. Extention of this model for the seven country case is very obvious.

Various aspects of policy spillovers and dynamic impacts of growth and development policies will be analysed further developing models.

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# 13 Example 3: Constitution, peace, growth and poverty alleviation in Nepal, May 2015-Sept 2015

The Constituent Assembly of Nepal (CAN) have thus far have failed to draft a democratic constitution for Nepal[3]. The major reason for this are believed to be an intense conflict among political parties holding different ideologies particularly regarding the root causes of poverty and inequality in Nepal that has increased substantially in last two decades. Failure to resolve the constitutional issues has resulted in weak government and lack of investment and employment opportunities. While the economy could not grow because of internal conflict and political instability, thousands of young people are forced to migrate abroad because of lack of employment prospects in the country. Economy has lost manpower that could actively contribute to the growth process. Nepal's performance has been dismal compared to neighboring countries in terms of per capita income either in market prices or in PPP terms.

This study has three major objectives:

[3]The CAN with 601 members was formed from the general election on May 28, 2008. It ran for four years till May 28, 2012 but failed to draft a constitution and was dissolved without a constitution. In July 2010 even seven rounds of elections could not elect a PM for the country, then a working coalition was formed but some progress was made in the peace process particularly in integrating the Maoist guerillas in the Nepalese Army. Then the another election took place for the CAN in November 2013 but it still is struggling to promulgate a constitution on the basic of consensus because of non-cooperation by the Maoist led coalition. Now the Napali Congress Party and the UML led coalition is making efforts to use its two third majority to pass a constitution. Maoists have threatened to boycott this constitution and cause further troubles.

1. (a) To measure the depth of poverty and pattern of inequality in the distribution of income in Nepal based on household level information at the moment.
   (b) To find a mechanism of cooperation among institutions or parties that can help alleviating poverty in Nepal within a shortest possible time period.
   (c) To provide a thorough comparison of successes and failures of poverty reduction strategies in South Asia, and possibly also from other regions.

**Literature review: Impact of Political Conflict on Poverty and Income Inequality**
Available documents on poverty from the National Planning Commission of Nepal, Ministry of Finance and Central Bureau of Statistics explaining the poverty situation in Nepal are old. Reports of international agencies such as the World Bank or the Asian Development Bank or WTO are helpful but very general and outdated particularly in the current context of conflict. Many macro and micro level studies in India, particularly in the context growth oriented policies of Modi government from May 2014, could be relevant but have not been properly accounted so far while analysing poverty and income inequality studies in Nepal.

A significant amount of work has been done on the issue of measurement and analysis of poverty using statistical, econometric and dynamic general equilibrium models for evaluation of micro and macro level policies over years. UK has a history of more than 100 years of study on basic need since Rowntree's (1899) studied the minimum living standard for a respectable life in York in Britain (see updates in Glennester, Huills, Piachaud and Webb (2004)), Orshansky (1965). As Atkinson (1970) argues "a poverty line cannot be defined in a vacuum, but only in relation to a particular society at a particular date". Accurate measurement of poverty and determining the appropriate methods for its reduction has been an issue of theoretical investigation in works of Sen (1976), Foster and Shorrocks (1985), Basu (1985), Vaughan (1987), Preston (1995), Shorrocks (1995) and Chakravarty (1997), Bhattarai (2010).

Traditionally measurement of poverty is expressed in terms of head-count ratios and the income-gap ratios. Inequality is studied in terms of Gini coefficients, standard deviations and Lorenz curve. Many empirical studies that have appeared recently aim to justify and monitor programmes aimed at reducing poverty using these concepts. Such poverty reduction strategy frameworks are often linked to the millennium development goals (UNDP (2000)). Earlier OECD (1976), UNDP (1991), World Bank (1991), Ravallion (1996) tried to assess depth of poverty in low income countries in Asian, Africa and Latin America.

It is still difficult to find appropriate models for alleviation of poverty that takes account of the structural features and provides the long run growth path based on the scientific analysis of micro and macroeconomic policy issues for Nepal.

Current political economic problem in Nepal is due to the failure of development of a mechanism for peace and alleviation of poverty. How does poverty feature in growth trajectories of major political parties? How would the distribution of income be affected by policies mentioned in their manifestos? This question has not been answered in Nepal. The current research project aims to fill this gap developing both theoretical and applied models for Nepal based on existing information on household surveys, time series data as well as creating a new database based on new surveys of households across various regions and districts and in various income groups. It will assess impacts of current conflict and public policies on prospects of majority of people in Nepal.

A comprehensive understanding of the issue requires:

1. Macro level studies that involve building suitable macroeconomic models required to analysis

of impacts of fiscal, monetary, exchange rate and trade policies on growth, income distribution and efficiency of allocation of resources in terms of welfare of households in the economy.

2. Micro survey will be conducted to take fresh information on income, consumption, labour supply, education, occupation, family and local or regional backgrounds of households. Study will cover Hill, Mountains and Tarai and provinces in the federal republic of Nepal.

**Cost of the project** This study is continuation of the current project "Conflict and Economic Growth in the Federal Republic of Nepal 2009-10", which was based on stratified random sampling technique with respondents over 55 districts of Nepal. Two students[4] in the MA programme in rural development in Kirtipur, Kathmandu Nepal did the field survey for the first phase of this study. The next phase of research project is appropriate for students willing to extend this project to analyse impacts of conflict on poverty and work in it for their Masters' dissertation. It will consists of two exercises; an up to date survey based analysis of poverty and modelling of economic policies to contain poverty. For the first part student-researcher will collect and process information on up to 500 households, and do preliminary literature review and analysis. Database will be created based on the CBS surveys if available, otherwise in fresh interviews . Statistical summary of all variables will be prepared using standard statistical software such as STATA or SPSS. Second aspect relates to dynamic modelling of poverty in Nepal. This model will be based on micro and macro realities of the Nepalese economy.

Findings will be presented in reports to be distributed to concerned public and private organisations in Nepal. Results will also be used to write good up to date journal articles. Each of two researchers will work for 6 months with a salary of Rs.20,000 a month; thus the salary cost amounts to Rs 240,000. Other expenses will be Rs. 60,000. Thus the total cost of the project will be Rs 300,000.

An academic from the Business School of the University of Hull, UK is willing to provide some supervision externally to the project. Hull economist(s) have published books and articles on analysis of poverty, economic development and economic modelling; details of this can be found in his web site at http://www2.hull.ac.uk/hubs/research.aspx or at http://www.hull.ac.uk/php/ecskrb/.

**Expected Outcomes** This research project expects to have following outcomes by the end of the project.

1. Measurement of the impact of conflict on poverty.
2. Analysis of the mechanism required for analysis of the depth of poverty and strategy for poverty alleviation and strengthening growth process in Nepal.
3. An analysis of the causes, consequences of existing inequality in income distribution and remedial measures to correct to gap between rich and poor.
4. International comparison of poverty alleviation strategies, particularly related to the South Asia.
5. Publication of report of the research project.

[4]Kedar Bhattarai and Suredra Pathak.

### 13.0.1 Survey Questionnaire (a simple example)

Problem Poverty Alleviation and Income Inequality in Nepal 2013

**I. General Background**

1. Age 2. Gender M F 3. Ethnic background:
4. Education 5. Profession 6. Average monthly income
7. District

.

**II. Specific questions**

1. What is your average monthly expenditure on following items?

Table 24: Average Monthly Expenditure

| | Expenditure in Rupees |
|---|---|
| Food | |
| Housing | |
| Clothes | |
| Education | |
| Health | |

2. What is your monthly income?

Table 25: Average Monthly Income

| | Income in Rupees |
|---|---|
| Wages | |
| Rent/Property | |
| Financial Assets | |
| Pension | |
| Other | |

3. Provide information on your family

Table 26: Number of family members at work

| | Number |
|---|---|
| Total | |
| At work | |
| At school | |
| Unemployed | |
| Outside the country | |

**III. Qualitative questions**

1. Given your education, professional skill and interest what is the most realistic option that you would have chosen if you had an access to finance it?

2. What is your ambition in life?

3. How much money do you need to get started on the work of your choice?

4. What can the national government do for you to achieve above target?

5. What can the local government do for you to achieve above target?

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## 13.1 Process of Dissertations

Dissertation done well can be academically rewarding and might generate confidence, motivation and satisfaction and a framework of thinking to link all pieces of work done in degree up to this point. It is more than individual essays you might have written for various modules. It should reflect your interest and professional knowledge in analysing an important issues using qualitative or quantitative framework whatever may be appropriate for you analysis. It must have clear aims and objectives and a set of questions and hypotheses to test cause-effect relationships among variables being studied. You must have clear idea on the relevant literature, methodology, expected results and a scientific approach to investigate the issue.

Think hard why the topics being studied is so important for dissertation, what are theoretical or empirical tools most appropriate for that topic, and what are the expected results. Above all think about your original contribution to the existing literature and analysis. Choose a topic in which you can make a significant contribution. Do not plunge on the primary or secondary data unless you are clear on these points and analytical techniques. Though these points must have been clear from lectures so far or from the research methods studied earlier, please pay close attention to following nine points:

1. Issue of research should be very specific, well defined and manageable within the time allowed for it. A good focus in the topic during the study is absolutely vital for a successful dissertation which is possible when the research question is very specific. Narrow it down. Quality of a dissertation improves by depth rather than its breadth. The depth of analysis counts more than its breadth.

2. Clarity on relevant theory applicable to the issue at hand is very important. A theory establishes links among a number of variables and helps to predict with some degree of confidence how a certain change in one variable can affect the value of another variable(s). Use familiar diagrams charts or equations for quantitative analysis or smooth out the flow of arguments that is generated from the skills learnt in other modules and reading relevant journal articles. More challenging dissertations may require even exploring methods based on critical review of the literature even though they have not been studied before.

3. Students are expected to assess, summarize, criticise, examine, question, challenge and digest the theoretical and empirical arguments and show originality by comparing and contrasting findings your own analyses and views regarding the issue under study to that in the literature. It is expected that students have a critical review of the theory and the literature relevant to the study. Consulting academic databases such as the JSTOR, SSRN, Econlit and with key words are very useful in finding out the most important works done in the area. Many journals have home pages with freely downloadable articles. Read literature critically with particular focus on theoretical structures, findings and limitations of the study.

4. Topics and issues should be introduced smoothly and gently in terms of complete and meaningful sentences. Use simple methodology that you thoroughly understand. More complicated specification of simulation or econometrics methods or surveys follow from simple logic. Which method is suitable and appropriate for a study depends partly on the objectives and questions of research as well as on one's beliefs and interest and skills. Whatever is being done should be done more professionally convincing readers about the major question of the study and how the research is being undertaken. In the past some did macroeconomic models, others did micro economic analyses and still others based their studies on surveys or data analyses.

5. It is very important to acknowledge ideas borrowed from the literature and maintain academic honesty by referencing works of other people. Follow rules of quotations and punctuations to avoid plagiarism. Write sentences in your own words and follow a standard format for dividing chapters sections and subsections. For a journal article and a book site as Ramsey (1928) or Keynes(1936) in the main text of dissertation and provide a complete list references for each source used as following:

   - Ramsey, F.P. (1928) A Mathematical Theory of Saving, Economic Journal 38, December, 543-559.
   - Keynes J. M. (1936) The General Theory of Employment, Income and Interest Rate, Cambridge University Press, London.

6. Give the reader of your dissertation an impression that you know what you are writing on. Explain each diagram, chart and equation clearly and discuss how it proves your points. Use relevant cross section, time series or survey data to be used for the study and plot their levels, shares, ratios, proportions, find correlations, regressions or simulations as appropriate to see the patterns and to convince yourself whether they support the theoretical reasoning or hypotheses. Extend the number of years, or enlarge the size of the sample if necessary.

7. Actual presentation of research results is very important. It is often found that some dissertations suffer because of poor writing though students might have done very good research. Poor writing can reduce marks substantially. Write complete and meaningful sentences. Check spelling and grammar and rules of punctuation and quotations. Express ideas in short sentences rather than long ones and avoid jargons as far as practicable. Write one idea in one paragraph. Make sure that arguments flow smoothly among paragraphs. Divide the study in sections in natural order of organisation and give some layout or plan of the study in the beginning so that any reader of the thesis can follow the whole presentation without any difficulty.

8. Edit the manuscript several times before submission. Do not hesitate to cut down irrelevant sections. Be consistent throughout the manuscript. Use summary rather than in extensive style to economise on space available.

9. It is the professional presentation and quality of work that matters in evaluation.

### 13.1.1 A Tentative Structure of a Student Research Report

1. (a) i. Introduction: Motivation to the Topic
      - Why is it necessary to study the chosen topic? What is original? What is known about it and what is unknown? More specifically what are the major objectives of the research? 5 pages

   ii. Review of the Literature
      - What are the most important works done in this area? Who have done it? What was their theoretical structure? How did they analyse the issue? What did they find? What are limitations of their study? How can the proposed study improve it? 8 pages

   iii. Methodology of the Study

- In what way is the proposed methodology different than in found in the literature? What are major assumptions and what are their limitations? What is the major structure of analysis? 10 pages

iv. Analytical structure: Main part

- Does the study involve a coherent mathematical model? Or does it contain applied work? How do different pieces of model structure fit together? What is the coherent and consistent story coming out of the analysis? How robust is the theoretical structure? 15 pages

v. Empirical support and application

- How can the model presented above applied in real life? What sorts of data does it use? If data were not in model consistent format what need to be done to make it consistent. How robust are the empirical results? 15 pages

vi. Major findings, Implications and Recommendations

- What were the major findings of the study? How much do they fulfil the objectives of the study? 5 pages

vii. References

- Journal Articles, books, Web pages. 2 pages

viii. Appendix

Survey questions, data series, charts, figures and other elements that support the main body of the dissertation.

**Relevant web pages:**
http://www.econometricsociety.org/; http://www.aeaweb.org/aer/index.php; http://www.res.org.uk/economic/ejbro
http://www.imf.org/external/pubs/ft/weo/2010/01/weodata/index.aspx;
http://www.ifs.org.uk/publications/789
http://www.esds.ac.uk/international/; http://www.bankofengland.co.uk/;
http://www.hm-treasury.gov.uk/
http://www.eea-esem.com/EEA/2010/Prog/ - look at fiscal policy sessions.
http://editorialexpress.com/conference/GAMES2012/program/GAMES2012.html
http://www.eea-esem.com/EEA-ESEM/2012/prog/list_sessions.asp

# 14 Econometric and Statistical Software

- Excel
- OX-GiveWin/PcGive/STAMP
- Eviews
- Shazam
- microfit
- RATS
- LIMDEP

- GAUSS
- STATA/SPSS
- **http://www.feweb.vu.nl/econometriclinks/;** https://www.aeaweb.org/rfe/
- http://www2.hull.ac.uk/acs/ict/software/software_sales.aspx
- http://www2.hull.ac.uk/student/studyadvice.aspx
- http://www2.hull.ac.uk/student/studyadvice/mathematicsresources.aspx

1. Excel Spreadsheets are very user friendly and could be used for algebraic calculations and statistical analyses for many kinds of economic models. First prepare an analytical solution by hand then use Excel formula to compute. Excel has constrained optimiser routine at tool/goal seek and solver command. It also contains matrix routines to get determinants of matrices and to multiply and invert them using multiple cell options. Koop (2007) is a brilliant text for analysis of economic data using excel. Koop G (2007) Analysis of Economic Data, Wiley, UK.

2. OX-GiveWin/PcGive/STAMP (www.oxmetrics.net) is a very good econometric software for analysing time series and cross section data. This software is available in all labs in the network of the university by sequence of clicks Start/applications/economics/givewin. Following steps are required to access this software.

a. save the data in a standard excel file. Better to save in *.csv format .

b. start give win at start/applications/economics/givewin and pcgive (click them separately)

c. open the data file using file/open datafile command.

d. choose PcGive module for econometric analysis.

e. select the package such as descriptive statistics, econometric modelling or panel data models.

d. choose dependent and independent variables as asked by the menu. Choose options for output.

e. do the estimation and analyse the results, generate graphs of actual and predicted series.

A Batch file can be written in OX for more complicated calculations using a text editor such as pfe32.exe. Such file contains instructions for computer to compute several tasks in a given sequence.

# References

[1] Doornik J A and D.F. Hendry (2009) OxMetrics Manual (PC-Give Volume I-III, GiveWin; G@RCH, STAMP),Timberlake Consultants Limited, London

## 14.1 Mathematical software

### 14.1.1 GAMS

4. GAMS is good particularly in solving linear and non-linear problems. It has widely been used to solve general equilibrium models with many linear or non-linear equations on continuous or discrete variables. It comes with a number of solvers that are useful for numerical analysis. For economic modelling it can solve very large scale models using detailed structure of consumption, production and trade arrangements on unilateral, bilateral or multilateral basis in the global economy where the

optimal choices of consumers and producers are constrained by resources and production technology or arrangements for trade.

It is a user friendly software. Any GAMS programme involves

- declaration of set, parameters, variables, equations,
- initialisation of variables and
- setting their lower or upper bounds and
- solving the model using Newton or other methods for linear or non-linear optimisation
- and reporting the results in tables or graphs (e.g. ISLM.gms ).

Full version of GAMS/MPSGE program is good for large scale standard general equilibrium models. GAMS programme can be downloaded from demo version of GAMS free from www.gams.com/download).

The check whether the results are consistent with the economic theory underlying the model such as ISLM-ASAD analysis for evaluating the impacts of expansionary fiscal and monetary policies. Use knowledge of growth theory to explain results of the Solow growth model from Solow.gms.

Consult GAMS and GAMS/MPSGE User Manuals, GAMS Development Corporation, 1217 Potomac Street, Washington D.C or www.gams.com or www.mpsge.org for GAMS/MPSGE.

**For other relevant software visit: http://www.feweb.vu.nl/econometriclinks/ or** https://www.aeaweb.org/rfe/;http://scholar.google.co.uk/

### 14.1.2 MATLAB

MATLAB is widely used for solving models. It has script and function files used in computations.

Both have *.m extensions. Its syntax are case sensivite. Solving a system of linear equations and handling matrices

Example 1

Write a programme file matrix.m like the following and try run.

```
% now solve a linear equation
% 5x1 + 2x2 =20
% 3x2 + 4x2 =15
k =[5 2;3 4];
n = [20 15];
kk = inv(k)
x = kk*n'
```

One more example of system of equation and factorisation of matrices

```
A=[1 2 3; 3 3 4; 2 3 3]
b=[1; 2; 3]
%solve AX=b
X = inv(A)*b
%eigen value and eigenvectors of A
[V,D]=eig(A)
%LU decomposition of A
[L,U]=lu(A)
%orthogonal matrix of A
```

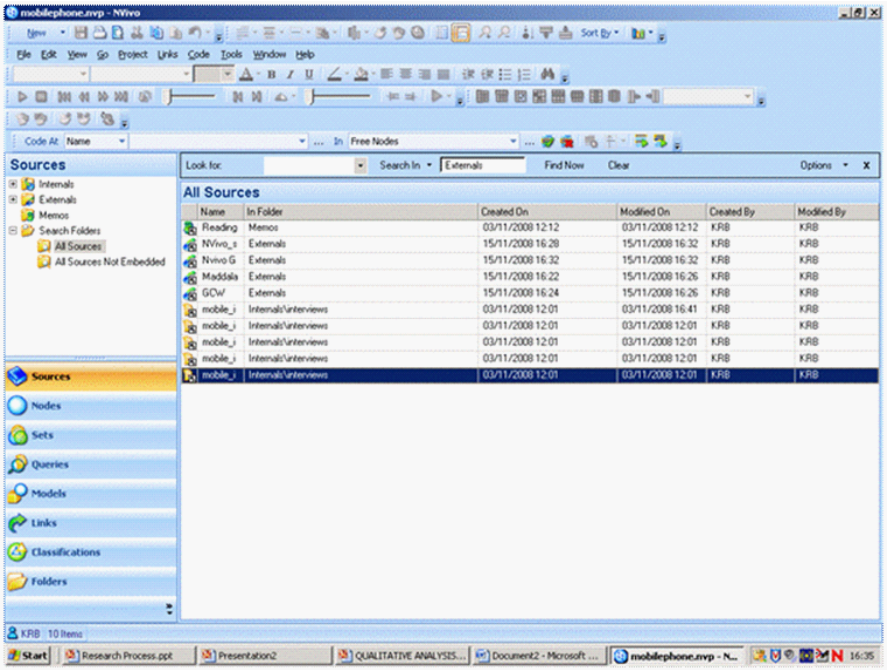
[Q,R]=qr(A)
%Cholesky decomposition (matrix must be positive definite)
%R = chol(A)
%Singular value decomposition
[U,D,V]=svd(A)
Contents.m for list of files in MATLAB demo. MATLAB demo available in http://www.youtube.com/.

## 14.2 Nvivo for Qualitative Data

Get NVivo from http://www.qsrinternational.com/

Steps in implementing an NVivo Project:

1. Start NVivo
2. Open a new project i.e. mobilephone.nvp ; make sure to save project frequently and close project before exiting NVivo.
3. Create subfolder in internal folder to keep interviews, field note, photos, videos in internal folder
4. Put other useful files in external folder
5. Put memo items, i.e. the list of files in memo
6. Create appropriate nodes (free, tree, cases, relationship and matrices)
7. Define sets useful for the study (buyers, sellers, markets, interviewer)
8. Create queries by words/ phrases to check occurrence of such key words.
9. Create static or dynamic models and choose appropriate shapes for models
10. Implement the models and determine if they support hypotheses.



### 14.3 Assignment:

Write a project proposal in a certain topic of your interest in about 2000 words. It should contain good motivation, literature review, clear methodology or models to be used, list or description of data sources for empirical estimation, computation or simulation, expected research outcomes and a list of references.

Be original, critical, systematic, concise, consistent, organised in presentation of your arguments.

The proposal should be typed in the double space, checked for spelling and grammar, with page numbers. Put word counts in the top right corner of the front page. You must read School policies regarding academic honesty and consequences of plagiarism as mentioned in the Business School Skills Handbook and declare academic honesty by filling in cover sheet of submission.

Literature search: Library subscribes to many academic journals that have good articles useful for an essay. These include Economics Letters, Applied Economies Letters, Journal of Development Studies, Macroeconomics Journal, National Institute's Bulletin of Economic Research, Oxford Economic Bulletin, Quarterly Bulletin of Bank of England and Yorkshire Bulletin. Construct your list of references using electronic databases such as the Econlit and the JOSTR available through the www.hull.ac.uk/lib/.

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# 15 Potential Topics for Research

There are millions of topics that might be interesting, see the hard or electronic copies of journals available through the library (JSTOR, Econlit, Business source premier, SSRN). Hundreds of thousands of economists have written so many things in so many topics and fields. A list can be generated as:

1. National Economies (UK, EU, US, G7, BRICS, LDCs, OECD, ASEAN, Asia, Africa, Americas)
   (a) Global economic and financial crisis
   (b) Energy and economy: security, environment and prices.
   (c) Stability of financial sector and economy
   (d) Fiscal and monetary policy rules for economic stability and growth
   (e) Trade, technology and growth at national and firm level
   (f) Reform in public policy: tax, spending, trade, regulations, redistribution
   (g) Can tax cuts finance budget deficit? (endogenous growth model)
   (h) Productivity growth in manufacturing and services sectors

(i) Human capital, research and development and economic growth

(j) Reasons for decline in public sector investment over years

(k) Role of public and private sector in funding of education and health sectors

(l) Impact of volatility of exchange and interest rate on exports

(m) Unemployment and inflation: in the long and the short run

(n) Two speed economy: growth of income of skilled and unskilled workers

(o) Provision for pension and social security

(p) Impact in the economy of rising oil and energy prices

(q) Liberalisation of the financial sector and private sector investment

(r) Regulations of market for certain products (e.g. carpets, mobile phones, banana, cars, cosmetics, drugs, cloths, furniture, nursing home, houses,)

(s) Arguments for and against privatisation of semi-public goods (i.e. railways, airlines, telecommunications)

(t) Determinants of wage and earning by professions, skills and regions

(u) Wage and income of sport clubs and top quality sport men and women

(v) Factors contributing to variation in growth of regional and local economies

(w) Equity and redistribution aspects of council tax, income tax or direct and indirect taxes

(x) Can growth occur with redistribution

(y) New deal and public and private sector partnership

(z) Role of demand side and supply side policies in the economy

() Patterns of consumption and saving by categories of households

() Employment and output multipliers with Input-output model of the UK economy

() Assessment of the reliability of macroeconomic forecasts

() London Stock Exchange and global economy

() Evaluation of economic costs and benefits of environmental levy

2. Economic Growth and Development Issues

(a) Why the four-fifth of the World is still underdeveloped?

(b) Why there is a North-South divide in per-capita income?

(c) Story of productivity growth: impact of industrial to internet revolutions

(d) Examination of poverty alleviation and economic growth

(e) How much can human capital contribute towards economic growth?

(f) Problems in transfer and adoption of technology

(g) Why cannot all countries grow at the same rate?

(h) What is the best technology to achieve higher rate of growth?

(i) Balanced versus unbalanced growth

(j) Does economic growth promote economic inequality?

(k) Do higher environmental standards reduce the rate of growth?

(l) Conflict, coalition and economic growth

(m) Economic costs of conflicts and HIV in Africa

(n) How much spending on research and development promote economic growth?

(o) Infrastructure and economic growth.

3. Macroeconomic issues

(a) Why are Keynesian models applicable more in some countries than in others?

(b) Why the rates of unemployment are higher in rigid labour markets?

(c) Examination economic problems when savings are not equal to investment

(d) Should households save more to make economy grow faster?

(e) Resource imbalances and economic crises

(f) First, second and third theories of economic crises.

(g) What are the best policy rules for stability and growth?

(h) Trade-off between unemployment and inflation?

(i) Can independent central banks do better than government controlled ones?

(j) How can exchange rate instability be harmful for an economy?

(k) Credibility of public policy and market reactions

4. Microeconomic issue

(a) Determinants of consumption and saving.

(b) Are consumers sovereign in market for goods and services?

(c) Income and substitution effects of price changes

(d) Analysis of short and long run cost of a certain firm or industry

(e) Consequences of factor and product taxes in a competitive market.

(f) Impacts of new technology in costs of production and supply.

(g) Market imperfections, inefficiency and regulation

(h) Is there any evidence for income and substitution effects in agriculture, manufacturing or engineering sectors?

(i) What are the welfare consequences of duopoly or oligopoly in the energy markets?

(j) Does deregulation and privatisation bring efficiency in allocation of resources?

(k) Analysis of expenditure pattern of households

(l) How elasticities of supply and demand affect the burden to a consumer?

(m) Examination of benefits and costs of privatisation

(n) Application of utility maximisation hypothesis under uncertainty.

5. Trade issues

   (a) Examination of Tariff and non-tariff barriers of trade
   (b) Does global free trade reduce or increase income and wage inequality?
   (c) Does the direct foreign investment promote economic growth?
   (d) Who benefits and who loses from regional economic cooperation?
   (e) Assessment of impact of increase in oil prices in the global income
   (f) Enlargement of EU and Economic prospects of its new members
   (g) How can liberal trade reduce pressure of illegal immigration to rich countries?
   (h) Evaluation of achievements of the WTO and the Doha rounds of trade talk
   (i) Leontief paradox or factor price equalisation?

6. Public Policy issues

   (a) Privatisation, efficiency and redistribution
   (b) market power and market structure
   (c) Should budget be balanced all the time?
   (d) Should government subsidise education or pay more unemployment benefit?
   (e) How can budget deficit create external and internal imbalances?
   (f) Can lower taxes reduce budget deficit?
   (g) Does the Ricardian equivalence apply in modern economies?
   (h) Examination of optimal tax rate and evidence
   (i) Optimal amount of public services?
   (j) Optimal allocation of public funds between local and central authorities

7. Households and labour market

   (a) Why current economic policies have created pension crisis in the West?
   (b) Income dynamics and life-cycle profiles of income
   (c) Determinants of wage and labour supply
   (d) Gender inequality in wage and earning
   (e) Link between educational qualification and earning
   (f) New technology, redundancy and structural transformation of labour market
   (g) Social safety net and unemployment: re-examination of Beveridge provisions.

8. Environment and natural resources

   (a) Economic impacts of Kyoto agreement
   (b) Consumption and production side externalities and social welfare
   (c) Double dividend hypothesis of environmental taxes

(d) Do tight environmental regulations reduce economic growth?

(e) Valuation and optimal use of non-renewable resources

9. Financial market and economy

   (a) Over or under investment, Value of a firm and the optimal stock of capital

   (b) Why banks tend to accumulate non-performing debt with weak monitoring?

   (c) Best way of financing economic development.

   (d) Analysis of risks and return in the financial market?

   (e) Volatility of financial markets and economy

   (f) Can Tobin tax (transaction cost) deter financial crisis?

10. Education economics

    (a) Impacts of universal primary education in skill formation

    (b) Who should pay tuitions: students or the government?

    (c) Matching education and job market

11. Country specific studies

    (a) Analysis of markets using microeconomic models

    (b) Model for macroeconomic policy evaluation and forecasting

    (c) Evaluation of impacts of tax reforms

    (d) Forecasting various policy scenarios

    (e) Movement in commodity prices and terms of trade

12. Commodity Markets:

    (a) agricultural goods: sugar, potato, cotton, rubber, green vegetables, tomato

    (b) fruit: apples, banana, pears, grapes, oranges, mango, jackfruit , coconut, nuts

    (c) grains: rice, corn, millet, wheat, maize, palm oil, peanuts

    (d) meat market: fish, beef, pork, lamb

    (e) drinks: wine, beer, whiskey, martini

    (f) metals and minerals: gold, silver, aluminium, steel, iron, copper, tin, zinc, oil

13. Uncertainty and asymmetric information

    (a) risks and uncertainty and markets for insurance

    (b) moral hazards and adverse selection

    (c) principle agent problem and monitoring

    (d) efficient contract and incentives

    (e) provisions for contingency

(f) Energy sectors
(g) Energy prices and trade in the global economy
(h) Generation and distribution of electricity and pollution
(i) Kyoto agreement on climate change
(j) Trade-off between trade and environment
(k) OPEC effect on oil and energy prices
(l) Renewable energy and exhaustion of non-renewable energy
(m) Fuel poverty
(n) Role of energy sector in the growth of economy
(o) Technological factors in promotion of the energy sector

# 16 Problems for the workshop

**Exercise 1: Economic Growth**

1. Define GDP, its growth rate, GDP per capita and growth rate of per capita GDP.
2. Represent macroeconomic balance in a standard equation.
3. Consider four set of countries in the global economy
   - Brazil, China, India, South Africa
   - Columbia, Ghana, Nigeria, Nepal
   - Germany, France, Japan, UK, USA
4. Download time series data for a period from 1980 to 2008 at least for two countries in each group on following key macro variables:
   - GDP (constant 2000 $)
   - Population
   - Consumption ratio (Household Final Consumption Expenditure as % of GDP)
   - Investment ratio (Gross fixed capital formation as % of GDP)
   - Public spending ratio (General government final consumption as % of GDP)
   - Export ratio (Export of goods and as % of GDP)
   - Import ratio (Imports of goods and as % of GDP)
   - Revenue ratio (Government revenue as % of GDP)
   - Budget balance (budget deficit as % of GDP)
   - Current account balance (as % of GDP)
5. Calculate the growth rate of GDP, per capita GDP and growth rates of per capita GDP for each country. Read your data file in GiveWin/PcGive.
6. Calculate correlations among ratios of consumption, investment, public spending, export, import and revenue.
7. Test whether variations in the growth rates of per capita output could be explained by above ratios.

**Exercise 2: Macro Economic Model**

1. What is a macroeconomic model? Consider a simple version of the IS-LM model.

Aggregate demand:

$$Y = C + I + G + X - M \tag{583}$$

Consumption function:

$$C = a + b(Y - T);\ a > 0, 0 < b < 1 \tag{584}$$

Investment:

$$I = I_0 - dr \tag{585}$$

Imports:

$$M = m_0 - m_1 Y + m_2 E \tag{586}$$

Money market:

$$\frac{MS}{P} = kY - nr \tag{587}$$

where $Y$ is the national income, $C$ the consumption, $I$ the investment, $r$ the rate of interest, $T$ the revenue from the lump sum tax, $G$ the government, $NX$ the net exports and E the exchange rate. The parameter a represents autonomous consumption, $b$ is marginal propensity of consume, $d$ the slope of investment function,$m_0$ and $m_1$ are the parameters in net export function and $I_0$ the autonomous investment. Here $Y, C, I$ and $NX$ are endogenous variables and $G, T, r, E$ are exogenous variables. MS is quantity of money and $P$ is the level of prices, $k$ and $n$ are parameters of money demand function. Fiscal policy in this model mainly involves raising revenue through lump sum taxes and spending it to provide public goods and services. Both of these could be used to stabilise the economy when taxes are proportional to income.

Estimation of this model requires additional data on following variables:

1. (a) i. Money supply
   ii. (GDP deflator
   iii. Annual growth rate of GDP
   iv. Population
   v. Employment
   vi. Gross national saving (constant 1995 $)
   vii. Official exchange rate (local currency per US $ period average)
   viii. Real interest rate (%)

   (b) Get these additional data for 1980-2008 period for the country chosen in the exercise 1.

   (c) Determine endogenous and exogenous variables. Get two reduced form equations one for goods and another for the money market.

   (d) Estimate the reduced form of these two equations using the simultaneous equation model option in GivWin/PcGive.

(e) Judge how well the model tracks the economy using historical simulations for this economy.

(f) Projection values of exogenous policy variable devising a simple policy rule (eg. government spending is to rise by 3 percent each year, exports to increase by 5 percent and the real money balance by 6%). Forecast values of endogenous variable of the model.

(g) Explain.

**Exercise 3: Growth with Redistribution**

1. Recent studies have generally concluded that economic growth does not automatically lower inequality; in many countries inequality has further increased despite impressive growth rates.

   (a) Discuss statistical properties of skewed distribution of income (mean, median, range, variance, skewness and kurtosis).

   (b) Show how the Lorenz curve and Gini coefficients could be used to show inequality of income in a certain year and to determine whether inequality has increased or reduced in a certain year compared given benchmark year.

   (c) Show how the non-linear Kuznet hypothesis on economic growth and inequality could be modelled using regression analysis.

   (d) Define the head count ratio and poverty gap often used in measuring poverty in empirical studies.

   (e) Economics data on income inequality is very scant and unsystematic. Construct the income distribution data for a household income groups as given below from the WDI database for any two years, e.g. 1980 and 2006.

   (f) Compute Gini coefficient. Compare Gini of two years and state whether inequality has increased or reduced based on above information.

   (g) Define poverty as half of the per capita income and determine the level of poverty by head count ratio, income gap ratio and Sen's index of poverty that takes both income gap and inequality into account.

   (h) Has trade increased or lowered poverty in this country?

**Exercise 4: Time series analysis**

1. What are the main properties of a stationary data series? Why is it important to check the stationarity of the data series before they are used in a regression ?

2. For one of the advanced countries chosen in exercise 1get quarterly data from the OECD data base on following variables:

   (a) GDP (constant 1995 $)

   (b) Consumption (Household Final Consumption Expenditure 1995 $)

   (c) Gross capital formation (constant 1995 $)

   (d) General government final consumption (constant 1995 $)

(e) Export of goods and services (constant 1995 $)

(f) Imports of goods and services (constant 1995 $)

(g) Government revenue

(h) Money supply

(i) GDP deflator

(j) Annual growth rate of GDP

(k) Population

(l) Employment

(m) Gross national saving (constant 1995 $)

(n) Official exchange rate (local currency per US $ period average)

(o) interest rate (%)

3. Determine order of integration of GDP, GDP components, inflation, interest rate and official exchange rate. Generate such result in a table.

4. Estimate a simple Keynesian consumption function and determine the existence of cointegration checking the stationarity of the residual errors.

**Exercise 5: Growth model with panel data**

1. Write the structure of panel data for 1..k number of variables for N number of countries and T time periods.

2. From the data you already have construct a panel data of economic growth, GDP, GDP income, ratios of consumption, investment, exports, government spending, inflation , the interest rate and the exchange rate, budget balance and deficit.

3. Estimate simple consumption function, net export and investment functions using pooled data; show time specific and country specific effects.

4. Examine the impacts of government budget balance or trade balance on growth rate of the economy.

**Exercise 6: Input-Output Model** Input-output model provides a snap-shot of the entire economy at a given period.

1. Formulate an input out model to determine the gross output of an economy.

2. Discuss backward and forward linkages of a 30 percent reduction in public expenditure.

3. How would revenue, tax and labour supply would linked to the gross output?

Table 27: Structure of an Input-Output Table

| | Primary | Secondary | C | I | G | X | M | Total |
|---|---|---|---|---|---|---|---|---|
| Primary | $X_{1,1}$ | $X_{1,2}$ | $C_1$ | $I_1$ | $G_1$ | $X_1$ | $M_1$ | $Y_1$ |
| Secondary | $X_{2,1}$ | $X_{2,2}$ | $C_2$ | $I_2$ | $G_2$ | $X_2$ | $M_2$ | $Y_2$ |
| Labour | $L_1$ | $L_2$ | | | | | | $wL$ |
| Labour tax | $t_{l,1}$ | $t_{l,2}$ | | | | | | $R_L$ |
| Capital | $K_1$ | $K_1$ | | | | | | $rK$ |
| Capotal tax | $t_{k,2}$ | $t_{k,2}$ | | | | | | $R_K$ |
| Total | $Y_1$ | $Y_2$ | C | I | G | X | M | |

**Exercise 7: Impacts of Trade and Exchange Rates**

1. Construct a Ricardian Trade model and estimate the production possibility frontiers for any two trading nations.
2. See the relation between the trade balance and exchange rates between these countries.
3. Test purchasing power parity theory of exchange rate.
4. Formulate the gravity model of trade for these two countries.
5. Review some models from the Global Trade Analysis Project (GTAP).

**Exercise 8: Impacts of Tax and Spending on households**

1. Consider demand and supply functions for two products, house and other goods or car and fuel.
2. Estimate these functions using available data and measure the deadweight loss effect of 20 percent increase in VAT.
3. What is the optimal size of the public sector? Discuss any three models that can be used to assess the impacts of taxes in output and employment.
4. Develop a computable general equilibrium tax model for a country of your choice and assess impacts of its current fiscal policy. Details of it depends on your interest (Germany, France, UK, USA, Nepal, India, China, Italy, Greece).
5.

### 16.0.1 Quality ranking of journals in Economics

Findings of theoretical and applied research are published in journals. Better the quality of a paper, more likelihood that it will be published in highly ranked journals, though this relationship is not always perfect one. It is instructive to look into the Association of Business School (ABS) ranking on quality of journals given below in process of reviewing the literature as well as in writing a paper.

**ABS 4* Journals** American Economic Review; Economic Journal; Econometrica; Journal of Labour Economics; Rand Journal of Economics; Journal of Political Economy; Journal of Monetary Economics; International Economic Review; Quarterly Journal of Economics; Review of Economic Studies; Journal of Econometrics; Journal of Economic Literature; Journal of Economic Perspective; Journal of Economic Theory; Journal of Economic Geography; Journal of Environmental Economics and Management; Journal of Financial Economics; Journal of Economic Perspective.

**ABS 3* Journals** Brookings Economics Papers; Journal of Economic Growth; Economic Letters; Econometric Theory; European Journal of Political Economy; European Economic Review; Journal of Development Economics; Canadian Journal of Economics; European Review of Agricultural Economics; Cambridge Journal of Economics; Journal of Applied Econometrics ; Journal of Comparative Economics; Journal of Development Studies;Journal of Economic Dynamics and Control; Journal of Health Economics; Journal of Economic Behaviour and Organisation; Journal of Economics and Management Strategy; Journal of Economics of Law and Organisation; Journal of Evolutionary Economics; Journal of Industrial Economics; Economica; Journal of Public Economics; Journal of European Economic Association; Journal of Urban Economics; Kyklos; Labour Economics; Ecological Economics; Land Economics; Oxford Bulletin of Economics and Statistics; Oxford Economics Papers;Oxford Review of Economic Policy; Review of Economics and Statistics; Review of International Economics;Social Choice and Welfare; Southern Economic Journal; World Bank Economic Review; Journal of International Economics..

**ABS 2* Journals** Advances in Econometrics; Agricultural Economics; Applied Economics; Applied Economics Letters; Annals of Public and Cooperative Economics; Applied Financial Economics; Australian Economic Review; Australian Journal of Agricultural and Resource Economics; Bulletin of Economic Research; Bulletin of Indonesian Economic Studies; Canadian Journal of Agricultural Economics; Contemporary Economic Policy; Contributions to the Political Economy; Defence and Peace Economics; Econometric Reviews; Economics of Education Review; Economics of Innovation and New Technology; Economics of Planning;Economics of Transition; Economist-Netherlands;Environmental Resource Economics; Fiscal Studies; Global Business and Economic Review; History of Political Economy; IMF Staff Papers; Insurance Mathematics and Economics; International Journal of Game Theory;International Journal of Economics of Business; International Review of Applied Economics; International Review of Economics and Finance; Journal of Agricultural and Resource. Economics; World Economy.

**ABA 1* Journals** Business Economics; Eastern European Economics; Economy and Society; Empirical Economics; Employee Relations Europe Asian Studies; Hitsubashi Journal of Economics; Information Economics and Policy;International Journal of Social Economics; Journal of Economic Methodology; Journal of Economic Psychology; Journal of Economics;Journal of Industry, competition and Trade; Journal of interdisciplinary Economics; Macroeconomic Dynamics.

For the latest version visit: http://www.associationofbusinessschools.org/node/1000257. Note also that there are many journals which have not been ranked by the ABS.

## 16.1 Best twenty articles in 100 years in the American Economic Review

Arrow, Kenneth J., B. Douglas Bernheim, Martin S. Feldstein, Daniel L. McFadden, James M. Poterba, and Robert M. Solow. 2011. "100 Years of the American Economic Review: The Top 20 Articles." American Economic Review, 101(1): 1–8.

1. Alchian, Armen A., and Harold Demsetz. 1972. "Production, Information Costs, and Economic Organization."American Economic Review, 62(5): 777–95.

2. Arrow, Kenneth J. 1963. "Uncertainty and the Welfare Economics of Medical Care." American Economic Review, 53(5): 941–73.

3. Cobb, Charles W., and Paul H. Douglas. 1928. "A Theory of Production." American Economic Review, 18(1): 139–65.

4. Deaton, Angus S., and John Muellbauer. 1980. "An Almost Ideal Demand System." American Economic Review, 70(3): 312–26.

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6. Diamond, Peter A., and James A. Mirrlees. 1971. "Optimal Taxation and Public Production I: Production Efficiency." American Economic Review, 61(1): 8–27.

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21. Shiller, Robert J. 1981. “Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?” American Economic Review, 71(3): 421–36.

## 16.2 Classics in Economics and the Economic Journal

1. Marshall Alfred (1893) Consumer’s Surplus, Annals of the American Academy of Political and Social Science, 3,Mar.,90-93

2. Marshall Alfred (1887) The Theory of Business Profits The Quarterly Journal of Economics, 1, 4, Jul., 477-481

3. Marshall Alfred (1969) SchoolThree Lectures on Progress and Poverty, Journal of Law and Economics, 12, 1, Apr., 184-226

4. Keynes J. M. (1937) The General Theory of Employment, Quarterly Journal of Economics, 51, 2, Feb., 209-223

5. Keynes J. M. (1943) The Objective of International Price Stability, Economic Journal, 53, 210/211, 185-187

6. Keynes J. M. (1924) Alfred Marshall, 1842-1924,Economic Journal, 34, 135, 311-372

7. Hicks John R. (1962) Economic Theory and the Evaluation of Consumers’ Wants, Journal of Business, 35, 3 , 256-263

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12. Spencer Peter D. (1984) The Effect of Oil Discoveries on the British Economy–Theoretical Ambiguities and the Consistent Expectations, Economic Journal, 94, 375, 633-644

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2. Jean-Charles Rochet and Jean Tirole (2003) Platform Competition in Two-Sided Markets" Journal of European Economic Association, 1:4:990-1029.

3. Daron Acemoglu, Philippe Aghion and Fabrizio Zilibotti (2006) Distance to Frontier and Economic Growth",Journal of European Economic Association, 4:1:37-74.

4. Alberto Alesina, Filipe R. Campante and Guido Tabellini (2008) Why is fiscal policy often procyclical?Journal of European Economic Association, 6:5:1006-1036.

5. Richard Blundell, Monica Costa Dias and Costas Meghir, (2004) Evaluating the employment impact of a mandatory job search program,Journal of European Economic Association, 2:4:569-606.

6. Ernst Fehr and John List,(2004) The hidden costs and returns of incentives—trust and trustworthiness among CEOs,Journal of European Economic Association, 2:5:743-771.

7. Jordi Galí, J. David López-Salido and Javier Vallés (2007) Understanding the effects of government spending on consumption, Journal of European Economic Association, 5:1:277-270.

8. Thomas Laubach New Evidence on the Interest Rate Effects of Budget Deficits and Debt, Journal of European Economic Association, 7:4:858-885.

9. James H. Stock and Mark W. Watson (2005) Understanding changes in international business cycle dynamics,Journal of European Economic Association, 3:5:968-1006.

10. Guido Tabellini (2010) Culture and institutions: economic development in the regions of Europe,Journal of European Economic Association, 8:4:677-716.

### 16.2.2 Best 40 articles in the Journal of Economic Perspectives

David Autor (2012) The Journal of Economic Perspectives at 100, Journal of Economic Perspectives, 26, 2,Spring, 3–18

1. Porter, Michael E.;van der Linde,Claas 1995 Toward a New Conception of the Environment-Competitiveness Relationship 9(4) 657

2. Kahneman, Daniel; Knetsch, Jack L.; Thaler, Richard H. 1991 Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias 5(1) 572

3. Diamond, Peter A.; Hausman, Jerry A. 1994 Contingent Valuation: Is Some Number Better than No Number? 8(4) 524

4. Fehr, Ernst; Gächter,Simon Fairness and Retaliation: The Economics of Reciprocity 2000 14(3) 490

5. Katz, Michael L.; Shapiro, Carl 1994 Systems Competition and Network Effects 8(2) 448

6. North, Douglass C. 1991 Institutions 5(1) 395

7. Koenker, Roger; Hallock, Kevin F. 2001 Quantile Regression 15(4) 375

8. Markusen, James R. 1995 The Boundaries of Multinational Enterprises and the Theory of International Trade 9(2) 375

9. Bernanke, Ben S.; Gertler, Mark 1995 Inside the Black Box: The Credit Channel of Monetary Policy Transmission 9(4) 365

10. Romer, Paul M. 1994 The Origins of Endogenous Growth 8(1) 365

11. Brynjolfsson, Erik; Hitt, Lorin M. 2000 Beyond Computation: Information Technology, Organizational Transformation and Business Performance14(4) 350

12. Nickell, Stephen 1997 Unemployment and Labor Market Rigidities: Europe versus North America 11(3) 344

13. Machina, Mark J. 1987 Choice under Uncertainty: Problems Solved and Unsolved 1(1) 338

14. Hanemann, W. Michael 1994 Valuing the Environment through Contingent Valuation 8(4) 332

15. Camerer, Colin; Thaler, Richard H. 1995 Anomalies: Ultimatums, Dictators, and Manners 9(2) 316

16. Ostrom, Elinor 2000 Collective Action and the Evolution of Social Norms 14(3) 313

17. Smith, James P. 1999 Healthy Bodies and Thick Wallets: The Dual Relation between Health and Economic Status 13(2) 311

18. Jarrell, Gregg A.; Brickley, James A.; Netter, Jeffry M. 1988 The Market for Corporate Control: The Empirical Evidence since 1980 2(1) 295

19. Andrade, Gregor; Mitchell, Mark; Stafford, Erik 2001 New Evidence and Perspectives on Mergers 15(2) 290

20. Scotchmer, Suzanne 1991Standing on the Shoulders of Giants: Cumulative Research and the Patent Law 5(1) 280

21. Simon, Herbert A. 1991 Organizations and Markets 5(2) 278

22. Bikhchandani, Sushil; Hirshleifer,David; and Welch, Ivo 1998 Learning from the Behavior of Others: Conformity, Fads, and Informational Cascades 12(3) 273

23. Elster, Jon 1989 Social Norms and Economic Theory 3(4) 272

24. Feenstra, Robert C. 1998 Integration of Trade and Disintegration of Production in the Global Economy 12(4) 272

25. Frank, Robert H.; Gilovich, Thomas; Regan, Dennis T. 1993 Does Studying Economics Inhibit Cooperation? 7(2) 272

26. Kirman, Alan P. 1992 Whom or What Does the Representative Individual Represent? 6(2) 272

27. Jensen, Michael C. 1988 Takeovers: Their Causes and Consequences 2(1) 268

28. Przeworski, Adam; Limongi, Fernando 1993 Political Regimes and Economic Growth 7(3) 268

29. Newhouse, Joseph P. 1992 Medical Care Costs: How Much Welfare Loss? 6(3) 265

30. Dixit, Avinash 1992 Investment and Hysteresis 6(1) 259

31. Oliner, Stephen D.; Sichel, Daniel E.2000 The Resurgence of Growth in the Late 1990s: Is Information Technology the Story? 14(4) 257

32. Cutler, David M; Glaeser, Edward L.; Shapiro, Jesse M. 2003 Why Have Americans Become More Obsese? 17(3) 250

33. Milgrom, Paul 1989 Auctions and Bidding: A Primer 3(3) 242

34. Portney, Paul R. 1994 The Contingent Valuation Debate: Why Economists Should Care 8(4) 239

35. Babcock, Linda; Loewenstein,George 1997 Explaining Bargaining Impasse: The Role of Self-Serving Biases 11(1) 231

36. Grossman, Gene M.; Helpman, Elhanan 1994 Endogenous Innovation in the Theory of Growth 8(1) 225

37. Palmer, Karen; Oates, Wallace E.; Portney, Paul R. 1995 Tightening Environmental Standards: The Benefi t-Cost or the No-Cost Paradigm 9(4) 222

38. Angrist, Joshua D.; Krueger, Alan B. 2001 Instrumental Variables and the Search for Identifi cation: From Supply and Demand to Natural Experiments 15(4) 221

39. Pritchett, Lant 1997 Divergence, Big Time 11(3) 209

40. Dawes, Robyn M.; Thaler, Richard H. 1988 Anomalies: Cooperation 2(3) 206

41. Lundberg, Shelly; Pollak, Robert A. 1996 Bargaining and Distribution in Marriage10(4) 206

42. Levitt, Steven D.; List, John A. 2007 What Do Laboratory Experiments Measuring Social Preferences Reveal about the Real World? 21(2) 174

43. Bebchuk, Lucian Arye;Fried, Jesse M.2003 Executive Compensation as an Agency Problem 17(3) 170

44. Kahneman, Daniel;Krueger, Alan B.2006 Developments in the Measurement of Subjective Well-Being 20(1) 165

45. Taylor, Alan M.; Taylor,Mark P.2004 The Purchasing Power Parity Debate 18(4) 164

46. Klemperer, Paul 2002 What Really Matters in Auction Design? 16(1) 157

47. Zitzewitz, Eric 2004 Prediction Markets Wolfers,18(2) 149

48. Mark 2005 The Impact of Social Structure on Economic Outcomes Granovetter, 19(1) 147

49. I Akerlof, George A.; Kranton, Rachel E. 2005 dentity and the Economics of Organizations 19(1) 120

50. Guiso, Luigi; Sapienza,Paola; Zingales,Luigi 2006 Does Culture Affect Economic Outcomes? 20(2) 120

51. Brunnermeier, Markus K. 2009 Deciphering the Liquidity and Credit Crunch 2007–2008 23(1) 111

52. Deaton, Angus 2008 Income, Health, and Well-Being around the World: Evidence from the Gallup World Poll 22(2) 46

53. Angrist, Joshua D.; Pischke, Jörn-Steffen 2010 The Credibility Revolution in Empirical Economics: How Better Research Design is Taking the Con out of Econometrics 24(2) 17

Make efforts to sort out best articles in other journals; JSTOR/econlit provide clues like most relevant, oldest, newest etc.

## 16.3 Articles by Hull Academics

University Innovations Site (also attend relevant public lectures):

- http://www2.hull.ac.uk/researchandinnovation.aspx
- http://www2.hull.ac.uk/researchandinnovation/researchareas.aspx
- See Business School Research Papers from
- http://www2.hull.ac.uk/hubs/research.aspx

### 16.3.1 Some Economics Papers from Hull Academics

Abbott Andrew , Joshy Easaw, Tao Xing (2008) Trade Integration and Business Cycle Convergence: Is the Relation Robust across Time and Space? Scandinavian Journal of Economics, 110, 2 , 403-417

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Ghosh Sugata, Andros Gregoriou (2008) The Composition of Government Spending and Growth: Is Current or Capital Spending Better? Oxford Economic Papers, 60, 3 , 484-516

Hammond C. J. (1991) Economic Computing in the 1990s; Recent Developments in Personal Computer HardwareThe Economic Journal, 101, 409 1611-1623

Hammond C. J. (2006) Factor Substitution in the Production of Library Services: Evidence from the North American Research Libraries, Managerial and Decision Economics, 27, 8 , 613-630.

Lancaster T (1979) Econometric Methods for Duration of Unemployment, Econometrica, 47:4:939-56.

Mills Terence C. , Gianluigi Pelloni and Athina Zervoyianni (1995) Unemployment Fluctuations in the United States: Further Tests of the Sectoral-Shifts,Review of Economics and Statistics, 77, 2, 294-304

Terry Williams, Colin Eden, Fran Ackermann and Andrew Tait (1995) The Effects of Design Changes and Delays on Project Costs, The Journal of the Operational Research Society, 46, 7, 809-818

Wilson J. S. G. (1949) Investment in a Monetary Economy, Economica, 16, 64, 321-335

# 17 Texts: Economics, Business, Finance, Management, Marketing Research

## References

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[2] Blanchard (2006) Macro Economics, Pearson.

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[13] Easterby-Smith M and R Thorpe and P Jackson (2011) Management Research, Sage, 4th edition.

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[17] Gibson W J and A. Brown (2009) Working with Qualitative Data, Save.

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[25] Mankiw G. N. (2007) Macroeconomics, 6th ed., New York : Worth.

[26] Mankiw N. G. and M. P. Taylor (2008) Macroeconomics: European Edition, Worth Publishers.

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[38] Stock J. H. and M.W. Watson (2007) Introduction to Econometrics, 2nd edition, Addison Wesley.

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## 17.1 A Tentative Schedule for Self Study on Research Methods

| Week | Topics |
|---|---|
| 1 | Setting research objectives and reviewing relevant literature |
| 2 | How to use data, statistics and regression in research papers (ESDS International/Datastream) |
| 3 | Basic Econometrics (Excel and Oxmetrics/PcGive/Eviews/STATA/SPSS) |
| 4 | Basic of time series and panel data models; Estimations/Interpretations. |
| 5 | Research Methods for applied micro and finance; GAMS/MATLAB |
| 6 | Methods for applied macroeconomic models |
| 7 | Input-Output Analysis (Matrix) |
| 8 | General Equilibrium Analysis: System Analysis |
| 9 | Game Theory: Nash Bargaining, Asymmetric information, principal agent (Strategic models) |
| 10 | Statistical analysis using large scale data (STATA/SPSS) |
| 11 | Linear and Non-Linear Programming (optimisation) |
| 12 | Writing and edition a proposal |